Math 307 - Spring 2016
Final Exam
June 8, 2016

Name: ______________________________

Student ID Number: ______________________________

- There are 8 pages of questions. In addition, the last page is the basic Laplace transform table. Make sure your exam contains all these pages.
- You are allowed to use a scientific calculator (**no graphing calculators and no calculators that have calculus capabilities**) and one **hand-written** 8.5 by 11 inch page of notes.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.** Give exact answers wherever possible.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- There may be multiple versions of the exam so if you copy off a neighbor and put down the answers from another version we will know you cheated. Any student found engaging in academic misconduct will receive a score of 0 on this exam. All suspicious behavior will be reported to the student misconduct board. In such an instance, you will meet in front of a board of professors to explain your actions.
- You have 110 minutes to complete the exam (13.75 minutes per page)

PAGE 1	10	
PAGE 2	10	
PAGE 3	15	
PAGE 4	12	
PAGE 5	17	
PAGE 6	12	
PAGE 7	12	
PAGE 8	12	
Total	100	

GOOD LUCK!

1. (10 pts) Find the explicit solution to $y' = 10e^y x \sin(x^2)$ with $y(0) = 0$.

2. (10 pts) One solution to $t^2y'' - 6y = 0$ is $y_1(t) = t^3$. Use reduction of order to find the general explicit solution.

3. (15 pts) The two parts below are NOT related.

(a) A home buyer gets a 30-year home loan for \$300,000 at 5% annual interest compounded continuously. Assume the buyer pays K dollars each month (so $12K$ dollars each year), where K is a constant. Let $y(t)$ be the balance of the loan after t years. As we did in class and in homework, model this situation by assuming the rate of change of the balance is equal to the yearly interest minus the yearly payments (where yearly interest is proportional to the balance with proportionality constant $r = 0.05$).
Give the differential equation and the two initial conditions. DO NOT SOLVE, just set up.

(b) The Logistic equation is the differential equation $\frac{dy}{dt} = ry\left(1 - \frac{1}{K}y\right)$, where r and K are positive constants. Solutions to this equation have been used to accurately model the size of some populations. (Your answers below might involved r and/or K)

i. Find and classify all equilibrium solutions for the Logistic equation as stable, semistable, or unstable.

ii. If $y = y(t)$ is the solution to the Logistic equation that also satisfies $y(0) = y_0$ with $0 < y_0 < K$, then what is $\lim_{t \to \infty} y(t)$?

4. (12 pts) A 16 liter vat initially contains 1 liter of pure water (no salt). Salt water containing 20 grams/liter of salt enters the vat at 3 liters/minute. The vat is well mixed and the mixture leaves the vat through a hole in the bottom at a constant rate of 1 liter/minute.
Let $y(t)$ be the amount of salt in the vat at time t minutes.
The differential equation is $\dfrac{dy}{dt} = A - \dfrac{y}{1+2t}$ with $y(0) = B$, where A and B are constants you should know from the description.

(a) From the description, what are the values of A and B?

(b) At the instant when the vat becomes full, how much salt will be in it?
(First, solve the *linear* differential equation).

5. (17 pts) **For all parts below**, the mass-spring system satisfies $u'' + \gamma u' + 4u = 0$, where γ is the damping constant. Distances are in inches and time is in seconds.

(a) Assume there is no damping and $u(0) = 1$ inches and $u'(0) = 6$ inches/second.
Give the largest value of $u(t)$. (Hint: First solve; your answer will be in inches)

(b) Assume there is no damping and the mass is set in motion. How long does it take for the mass to go from its lowest point back to its lowest point again?

(c) If the system is critically damped, give the general explicit solution.

(d) If $\gamma = \sqrt{7}$ lbs/(in/sec), give the general explicit solution.

6. (12 pts) Give the general explicit solution to $y'' + y' - 2y = 4t - 30\sin(t)$.

7. (12 pts)

(a) Give the Laplace transformation of $f(t) = t^3 + e^{-7t} + (t+4)u_6(t)$

(b) Give the inverse Laplace transformation of $F(s) = \dfrac{3s+2}{(s-1)^2+4} + \dfrac{e^{-2s}}{s-5}$

8. (12 pts) You make a cup of tea with an initial temperature of 200°F, which is too hot. You place it in a 30°F freezer for 0.2 hours (12 minutes), then take it out and set it on a table where room temperature is 70°F. Assume Newton's law of cooling with a cooling constant of $k = 1$, we get

$$\frac{dy}{dt} = -(y - y_s(t)), \text{ where } y_s(t) = \begin{cases} 30 & , 0 \leq t < 0.2; \\ 70 & , t \geq 0.2, \end{cases}$$

where $y = y(t)$ is the temperature of the tea after t hours.
Using Laplace transforms, solve for the function $y(t)$ **and**
give the temperature of the tea 20 minutes ($t = 1/3$ hours) after you made it, to the nearest degree.

Laplace Transform Table for Final Exam - Dr. Loveless

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1	$\frac{1}{s}$
e^{at}	$\frac{1}{s-a}$
$\cos(bt)$	$\frac{s}{s^2+b^2}$
$\sin(bt)$	$\frac{b}{s^2+b^2}$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$
t^n	$\frac{n!}{s^{n+1}}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$u_c(t)$	$\frac{e^{-cs}}{s}$
$u_c(t)f(t-c)$	$e^{-cs}F(s)$