Your Name

Student ID #

- You are allowed one two-sided handwritten notesheet for this midterm. You may use a scientific calculator; graphing calculators and all other course-related materials may not be used.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Give your answers in exact form (for example $\frac{\pi}{3}$ or $e^{-5\sqrt{3}}$) unless explicity stated otherwise by the question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 5 pages, plus this cover sheet. Please make sure that your exam is complete.
- You have 50 minutes to complete the exam.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

1. (10 total points) Find the expicit general solutions to the following first-order differential equations. Your answer should be a function in the form $y = g(x, C)$, where C is an integration constant parameterizing the family of solutions to the DE.

   (a) (5 points) $\frac{dy}{dx} - 2xy - x = 0$

   (b) (5 points) $\frac{dy}{dx} = \frac{y^2 - y}{x^2 + 1}$

2. (10 total points) Solve the following initial value problems. Your answer should be in the form $y = g(t)$, where there is no undetermined constant in g.

   (a) (5 points) $\frac{dy}{dt} = \frac{1-3ty}{t^2}, \quad y(1) = 0.$

   (b) (5 points) $\frac{dy}{dt} = e^{2t-3y}, \quad y(1) = 2.$

3. (10 points) The differential equation

$$\frac{dy}{dx} = \frac{2y+3x}{2y+2x}$$

is neither linear nor separable. Use the homogenizing substitution $v = \frac{y}{x}$ to find the solution to the DE with the initial condition $y(1) = 0$. Along with your answer state explicitly on what interval the solution is defined.

4. (10 total points) A reservoir on a farm initially contains 10000 liters of water, in which 200kg nitrate fertilizer is dissolved. The owner of the reservoir decides the amount of dissolved nitrate needs to be increased, so starts pumping in a 1kg nitrate:1 liter water solution at a rate of 100 l/min. However, the reservoir simultaneously develops a leak, and starts draining at a rate of 200 l/min.

   (a) (7 points) Assuming the solution remains perfectly mixed at all times, find the mass of nitrate in the reservoir at time t.

   (b) (3 points) What is the maximum amount of nitrate in the reservoir, and when does it occur?

5. (10 total points) We've seen in class and the homework a model for objects under free fall in which we assume the drag force acting on an object is proportional to its velocity. In reality a more accurate model is one in which drag force is proportional to the velocity *squared*. As such, a differential equation to model the velocity $v(t)$ of a falling object near the earth's surface is

$$\frac{dv}{dt} = -g + kv^2,$$

where g is acceleration due to gravity, and the proportionality constant k depends on the mass and aerodynamics of the object being dropped as well as atmospheric conditions.

(a) (5 points) For skydivers in freefall under standard atmospheric conditions, it's known that k is approximately $2.73 \times 10^{-3}\, m^{-1}$ (i.e. when working in metric k has units of inverse meters). Using this value and taking $g = 9.81\, ms^{-2}$, compute the terminal (limiting) velocity of such a skydiver. You may use decimal approximations in your final answer (but keep at least 4 digits precision at all points).

(b) (5 points) Use Euler's method with a step size of $h = 0.5$ to estimate a skydiver's downward velocity 1 second after jumping from a plane. You may use decimal approximations in your final answer (but keep at least 4 digits precision at all points).