## Partial Derivatives Quick Overview

In Math 307, we sometimes see functions of the form $f(x,y)$. This is called a multivariable function. It gives a third value, let's say $z$, for each valid value pair of values $(x,y)$ (that is $z = f(x,y)$). In Math 126, you will spend several weeks introducing and studying such functions. In this course, we will have a few occasions where we need to find a rate of change with respect to one of the variables. We will define:

$$\frac{\partial f}{\partial x}(x,y) = f_x(x,y) = \text{`the partial derivative of } f \text{ with respect to } x\text{'}.$$
$$\frac{\partial f}{\partial y}(x,y) = f_y(x,y) = \text{`the partial derivative of } f \text{ with respect to } y\text{'}.$$

For this course, you only need to know how to compute simple partial derivatives of functions of the form $f(x,y)$.

Here is how you compute $\frac{\partial f}{\partial x}$:
Treat everything in $f(x,y)$ as a CONSTANT except $x$ (*i.e.* treat $y$ like a constant). Then take the derivative with respect to $x$.

Here is how you compute $\frac{\partial f}{\partial y}$:
Treat everything in $f(x,y)$ as a CONSTANT except $y$ (*i.e.* treat $x$ like a constant). Then take the derivative with respect to $y$.

A few basic examples:

1. If $f(x,y) = x^3 + 2y^5 + 4$, then the partial derivatives are

   $\frac{\partial f}{\partial x} = 3x^2$ Note: $y$ is a constant so the deriv. of $2y^5$ is zero.

   $\frac{\partial f}{\partial y} = 10y^4$ Note: $x$ is a constant so the deriv. of $x^3$ is zero.

2. If $f(x,y) = x^4y^3 + 8x^2y + y^4 + 5x$, then the partial derivatives are

   $\frac{\partial f}{\partial x} = 4x^3y^3 + 16xy + 5$ Note: 8 and $y$ are coefficients of $x^2$, where $y^4$ is just a constant.

   $\frac{\partial f}{\partial y} = 3x^4y^2 + 8x^2 + 4y^3$ Note: $8x^2$ is the coefficient of $y$ and the deriv. of $y$ is 1.

3. If $f(x,y) = \frac{x^2}{y^3} = \frac{1}{y^3}x^2 = y^{-3}x^2$, then the partial derivatives are

   $\frac{\partial f}{\partial x} = \frac{2x}{y^3}$ Note: No need for quotient rule, only an $x$ in the numerator.

   $\frac{\partial f}{\partial y} = -3y^{-4}x^2$ Note: Again, no need for quotient rule, only a $y$ in the denominator.

4. If $f(x,y) = (x^2 + y^3)^{10} + \ln(x)$, then the partial derivatives are

   $\frac{\partial f}{\partial x} = 20x(x^2 + y^3)^9 + \frac{1}{x}$ Note: We used the chain rule on the first term.

   $\frac{\partial f}{\partial y} = 30y^2(x^2 + y^3)^9$ Note: Chain rule again, and second term has no $y$.

5. If $f(x,y) = xe^{xy}$, then the partial derivatives are

   $\frac{\partial f}{\partial x} = e^{xy} + xye^{xy}$ Note: Product rule, and chain rule in the second term.

   $\frac{\partial f}{\partial y} = x^2e^{xy}$ Note: No product rule, but we did need the chain rule.