NAME: SOL'N

Test Prep 3

Here are problems where you can practice undetermined coefficients. If you finish this page, try the problems on the back (for an extra point). You have 15 minutes.

1. Find the full solution to $y'' + 9y = 7\cos(4t)$ with $y(0) = 0$ and $y'(0) = 0$ (solve and find all constants).

[I] $r^2 + 9 = 0 \Rightarrow r = \pm 3i \Rightarrow y_1 = \cos(3t)$, $y_2 = \sin(3t)$

[II] $Y(t) = A\cos(4t) + B\sin(4t)$ · 9

$Y'(t) = -4A\sin(4t) + 4B\cos(4t)$ · 0

$Y''(t) = -16A\cos(4t) - 16B\sin(4t)$ · 1

$Y'' + 9Y \stackrel{?}{=} 7\cos(4t)$

$(9A - 16A)\cos(4t) + (9B - 16B)\sin(4t) \stackrel{?}{=} 7\cos(4t)$

$-7A \stackrel{?}{=} 7$ $\quad -7B \stackrel{?}{=} 0$

$A = -1$ $\quad B = 0$

$y' = -3c_1\sin(3t) + 3c_2\cos(3t) + \sin(4t)$

$y(t) = c_1\cos(3t) + c_2\sin(3t) - \cos(4t)$

$y(0) = 0 \Rightarrow c_1 + 0 - 1 = 0 \Rightarrow c_1 = 1$

$y'(0) = 0 \Rightarrow 0 + 3c_2 + 0 = 0 \Rightarrow c_2 = 0$

$$\boxed{y(t) = \cos(3t) - \cos(4t)}$$

2. What choice would you make for the form of a particular solution to $y'' + 9y = 7\cos(3t)$? (For this part, do NOT solve, just write down the form of a particular solution with A, B, C, *etc* for the constants and do NOT solve for A, B...)

$$\boxed{Y_p(t) = At\cos(3t) + Bt\sin(3t)}$$

ASIDE: WITH $y(0) = 0$ AND $y'(0) = 0$ YOU GET $c_1 = 0$, $c_2 = 0$, $A = 0$, $B = 7/6$

SO $y(t) = \frac{7}{6} t\sin(3t)$

PART OF AN OLD EXAM PROBLEM: A certain car has mass 800 kg and the combined effect of the springs in the suspensions system gives a spring constant of 16000 N/m. Your job is to design a damping mechanism which eliminates oscillations when the automobile hits a bump. What is the minimum value that the damping constant, γ, needs to be in order to eliminate oscillations?

$m = 800, \quad k = 16000$

$800u'' + \gamma u' + 16000u = 0$

WANT $\gamma^2 - 4mk = 0$ (CRITICALLY DAMPED)

$\gamma = \sqrt{4mk} = 2\sqrt{mk} = 2\sqrt{800 \cdot 16000} = 3200\sqrt{5}$

≈ 7155.4175

$\gamma =$ 7155 N/(m/s)

Questions for your instruction about the current material?

LET ME KNOW

MORE ON TEST PREP 3

FOR ANY ω OTHER THAN $\omega_0 = 3$ (i.e. $\omega \neq 3$)

$$y'' + 9y = 7\cos(\omega t)$$

WILL HAVE PARTICULAR SOL'N

$$y(t) = A\cos(\omega t) + B\sin(\omega t) \qquad \cdot 9 \quad y$$
$$y'(t) = -\omega A\sin(\omega t) + \omega B\cos(\omega t) \qquad \cdot 0 \quad y'$$
$$y''(t) = -\omega^2 A\cos(\omega t) - \omega^2 B\sin(\omega t) \qquad \cdot 1 \quad y''$$

$$y'' + 9y \overset{?}{=} 7\cos(\omega t)$$

$$-\omega^2 A\cos(\omega t) - \omega^2 B\sin(\omega t) + 9A\cos(\omega t) + 9B\sin(\omega t) \overset{?}{=} 7\cos(\omega t)$$

$$(9-\omega^2)A\cos(\omega t) + (9-\omega^2)B\sin(\omega t) = 7\cos(\omega t)$$

$$\Rightarrow (9-\omega^2)A = 7 \Rightarrow A = \frac{7}{9-\omega^2}$$

$$(9-\omega^2)B = 0 \Rightarrow B = 0$$

$$y = c_1\cos(3t) + c_2\sin(3t) + \frac{7}{9-\omega^2}\cos(\omega t)$$

($7 \leftarrow F_0$; $9 \leftarrow \omega_0^2$)

AND if $y(0) = 0$ AND $y'(0) = 0$

then

$$c_1 = -\frac{7}{9-\omega^2} \qquad c_2 = 0$$

NOTE: $\frac{7}{9-\omega^2}$ INCREASES WITHOUT BOUND AS $\omega \to 3$!

$$y = \frac{-7}{9-\omega^2}\cos(3t) + \frac{7}{9-\omega^2}\cos(\omega t)$$

$$y = \frac{7}{9-\omega^2}\left[\cos(\omega t) - \cos(3t)\right]$$

($F_0 \to 7$; $\omega_0^2 \to 9$; $3 \leftarrow \omega_0$)

} GENERAL SOL'N FOR ANY $\omega \neq 3$

GRAPHING TOOLS (TWO WAVE TRIG IDENTITIES)

$$\cos(A) + \cos(B) = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{B-A}{2}\right)$$

$$\cos(A) - \cos(B) = 2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{B-A}{2}\right)$$

$$\sin(A) + \sin(B) = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{B-A}{2}\right)$$

$$\sin(-x) = -\sin(x) \qquad \cos(-x) = \cos(x)$$

So

SINCE $\sin(x) = -\sin(-x)$ CAN FLIP IF YOU WANT

$$\cos(\omega t) - \cos(\omega_0 t) = 2\sin\left(\frac{\omega+\omega_0}{2}t\right)\sin\left(\frac{\omega_0-\omega}{2}t\right)$$

$$= -2\sin\left(\frac{\omega+\omega_0}{2}t\right)\sin\left(\frac{\omega-\omega_0}{2}t\right)$$