1. (11 pts)

5 (a) Find parametric equations for the line of intersection of the two planes $2x - y + 8z = 14$ and $2x - 2y + 4z = 2$.

$2x = 14 + y - 8z$

COMBINING $\Rightarrow$ $y + 4z = 12$

ONE POINT: $y = 0 \Rightarrow z = 3 \Rightarrow 2x = 14 + 0 - 8(3) = -10 \Rightarrow x = -5$

$P(-5, 0, 3)$

ANOTHER POINT: $z = 0 \Rightarrow y = 12 \Rightarrow 2x = 14 + 12 - 8(0) = 26 \Rightarrow x = 13$

$Q(13, 12, 0)$

$\vec{PQ} = \langle 18, 12, -3 \rangle$

$$\boxed{\begin{aligned} x &= -5 + 18t \\ y &= 0 + 12t \\ z &= 3 - 3t \end{aligned}}$$

MANY CORRECT ANSWERS

① $(x_0, y_0, z_0)$ NEEDS TO BE ON BOTH PLANES

② DIRECTION VECTOR MUST BE PARALLEL TO $\langle 18, 12, -3 \rangle$

3 (b) Consider the curve $y = 10 + 4x - x^2$ at $(x, y) = (3, 13)$.

i. Find a vector, $\mathbf{v}$, that has length 4 and is parallel to the tangent line to $y = 10 + 4x - x^2$ at $x = 3$.

$y' = 4 - 2x$

$y'(3) = 4 - 2(3) = -2$

SO $\langle 1, -2 \rangle$ IS PARALLEL TO THE TANGENT

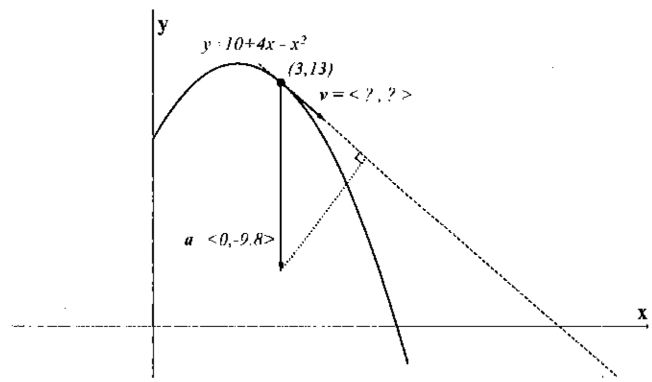
$$\boxed{\frac{4}{\sqrt{5}} \langle 1, -2 \rangle = \left\langle \frac{4}{\sqrt{5}}, \frac{-8}{\sqrt{5}} \right\rangle}$$

OR

$$\boxed{-\frac{4}{\sqrt{5}} \langle 1, -2 \rangle = \left\langle -\frac{4}{\sqrt{5}}, \frac{8}{\sqrt{5}} \right\rangle}$$

3 ii. Find the length of the projection of $\mathbf{a} = \langle 0, -9.8 \rangle$ onto $\mathbf{v}$, from part (i).

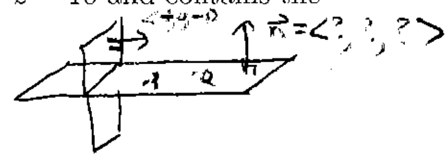
$$\text{comp}_{\vec{v}} \langle 0, -9.8 \rangle = \frac{0 + \frac{4}{\sqrt{5}} \cdot 19.6}{|\vec{v}| \leftarrow 4} = \boxed{\frac{19.6}{\sqrt{5}} \approx 8.765}$$




IF YOU USE $\vec{v} = \left\langle -\frac{4}{\sqrt{5}}, \frac{8}{\sqrt{5}} \right\rangle$, THEN YOU GET $-\frac{19.6}{\sqrt{5}}$ WHICH IS ALSO ACCEPTED IF IT MATCHES YOUR CHOICE OF $\vec{v}$.

2. (14 pts)

4 (a) Find the equation for the plane that is orthogonal to the plane $4x - z = 10$ and contains the points $P(3,2,3)$ and $Q(4,5,1)$.




$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 0 & -1 \\ 1 & 3 & -2 \end{vmatrix}$$

PARALLEL TO DESIRED PLANE $\to \langle 4, 0, -1 \rangle$

$\to \vec{PQ}\ \langle 1, 3, -2 \rangle$

$= (0 - -3)\vec{i} - (-8 - -1)\vec{j} + (12 - 0)\vec{k}$

$= \langle 3, 7, 12 \rangle = \vec{n}$

$$\boxed{3(x-3) + 7(y-2) + 12(z-3) = 0}$$

$$3x - 9 + 7y - 14 + 12z - 36 = 0$$

$$3x + 7y + 12z = 59$$

4 (b) Find an equation for the surface consisting of all points $(x, y, z)$ such that the distance from $(x, y, z)$ to $(0, 0, 2)$ is equal to the distance from $(x, y, z)$ to the $xy$-plane. **AND** give the precise name for this surface (Hint: Expand/Simplify your equation!)

$$\boxed{\begin{aligned} \sqrt{x^2 + y^2 + (z-2)^2} &= z \\ x^2 + y^2 + \cancel{z^2} - 4z + 4 &= \cancel{z^2} \\ x^2 + y^2 + 4 &= 4z \end{aligned}}$$

TRACES: $x = k \to$ PARABOLAS

$y = k \to$ PARABOLAS

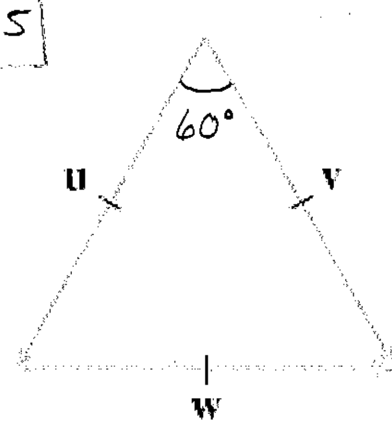
$z = k \to$ CIRCLES

NAME: CIRCULAR PARABOLOID

6 (c) In the picture below, **u**, **v**, and **w** are all vectors of length 3 (*i.e.* $|\mathbf{u}| = |\mathbf{v}| = |\mathbf{w}| = 3$). The vectors form an equilateral triangle (as shown). Using this information and important facts from class, find the following values:

i. $\mathbf{u} \cdot \mathbf{v} = |\vec{u}||\vec{v}|\cos(\overset{60°}{\theta}) = 3 \cdot 3 \cdot \frac{1}{2} = \boxed{\frac{9}{2} = 4.5}$

ii. $|2\mathbf{u} + 2\mathbf{w}| = 2|\vec{u} + \vec{w}| = 2|\vec{v}| = 2 \cdot 3 = \boxed{6}$




iii. the area of the triangle =

$= \frac{1}{2}|\vec{u} \times \vec{v}| = \frac{1}{2}|\vec{u}||\vec{v}|\sin \overset{60°}{\theta}$

$= \frac{1}{2} 3 \cdot 3 \frac{\sqrt{3}}{2} = \boxed{\frac{9}{4}\sqrt{3}}$

3. (13 pts) For ALL parts below, consider the curve, $C$, given by $x = 5 - t$, $y = t$, $z = t^2 - 10$.

4 (a) Find the **two** points $(x, y, z)$ where the curve, $C$, intersects the cylinder $x^2 + y^2 = 13$.

$$(5-t)^2 + t^2 \stackrel{?}{=} 13$$
$$\Rightarrow 25 - 10t + t^2 + t^2 = 13$$
$$\Rightarrow 2t^2 - 10t + 12 \stackrel{?}{=} 0$$
$$\Rightarrow t^2 - 5t + 6 \stackrel{?}{=} 0$$
$$\Rightarrow (t-2)(t-3) \stackrel{?}{=} 0$$

$t = 2 \Rightarrow \boxed{(x,y,z) = (3, 2, -6)}$

$t = 3 \Rightarrow \boxed{(x,y,z) = (2, 3, -1)}$

3 (b) Find parametric equations for the tangent line, $L$, to the curve, $C$, at $t = 1$.

$x(1) = 4$
$y(1) = 1$
$z(1) = -9$

$x' = -1$, $y' = 1$, $z' = 2t$ $\Rightarrow$ $x'(1) = -1$, $y'(1) = 1$, $z'(1) = 2$

$$\boxed{\begin{aligned} x &= 4 - u \\ y &= 1 + u \\ z &= -9 + 2u \end{aligned}}$$

6 (c) Consider a different line $L_2$ given by $x = -2 + 6u$, $y = 2 + 4u$, and $z = 5 + 2u$. This line, $L_2$, and the curve, $C$, intersect in one point. Find the angle of intersection (round your answer to the nearest degree).

① $5 - t \stackrel{?}{=} -2 + 6u$
② $t \stackrel{?}{=} 2 + 4u$
③ $t^2 - 10 \stackrel{?}{=} 5 + 2u$

COMBINE: $5 - (2+4u) \stackrel{?}{=} -2 + 6u$
$3 - 4u = -2 + 6u$
$5 = 10u$
$u = 1/2 \Rightarrow t = 2 + 2 = 4$

CHECK $4^2 - 10 = 6 \stackrel{\checkmark}{=} 5 + 2(\frac{1}{2})$

TANGENT VECTORS: $\langle -1, 1, 2t \rangle$ at $t = 4 \Rightarrow \langle -1, 1, 8 \rangle = \vec{a}$
$\langle 6, 4, 2 \rangle$ at $u = \frac{1}{2} \Rightarrow \langle 6, 4, 2 \rangle = \vec{b}$

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$$

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = \frac{-6 + 4 + 16}{\sqrt{66}\sqrt{36+16+4}}$$

$$\theta = \cos^{-1}\left(\frac{14}{\sqrt{66}\sqrt{56}}\right) \approx 76.69^\circ \approx \boxed{77^\circ} \approx 1.34 \text{ RADIANS}$$

4. (12 pts) For ALL parts below, consider the curve given by the position function $\mathbf{r}(t) = \langle t^3, 3t^2, 6t \rangle$.

2 (a) Multiple Choice (Circle ALL that are true, there may be more than one):
Every point on the curve is also on the surface:

Circle ALL that true: (i) $18x = yz$ (ii) $y^2 + z^2 = 1$ (iii) $12y = z^2$ (iv) $y - z = 0$

5 (b) Find the curvature, $\kappa$, of $\mathbf{r}(t)$ at $t = 0$. (Reminder: You don't need to find the general formula, only the value at $t = 0$.)

$$\vec{r}\,'(t) = \langle 3t^2, 6t, 6 \rangle \Rightarrow \vec{r}\,'(0) = \langle 0, 0, 6 \rangle$$

$$\vec{r}\,''(t) = \langle 6t, 6, 0 \rangle \Rightarrow \vec{r}\,''(0) = \langle 0, 6, 0 \rangle$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 6 \\ 0 & 6 & 0 \end{vmatrix}$$

$$\vec{r}\,'(0) \times \vec{r}\,''(0) = (0-36)\vec{i} - (0-0)\vec{j} + (0-0)\vec{k}$$
$$= \langle -36, 0, 0 \rangle$$

$$\frac{|\vec{r}\,'(0) \times \vec{r}\,''(0)|}{|\vec{r}\,'(0)|^3} = \frac{36}{(6)^3} = \boxed{\frac{1}{6}} \approx 0.1\overline{6}$$

ASIDE: RADIUS OF CURVATURE = 6

5 (c) Find the distance (arc length) along the curve $\mathbf{r}(t)$ from the point $(0, 0, 0)$ to $(1, 3, 6)$.

$(0,0,0) \leftrightarrow t = 0$ $\qquad (1,3,6) \leftrightarrow t = 1$

$$\int_0^1 \sqrt{(3t^2)^2 + (6t)^2 + (6)^2}\, dt = \int_0^1 \sqrt{9t^4 + 36t^2 + 36}\, dt$$
$$= \int_0^1 \sqrt{9(t^4 + 4t^2 + 4)}\, dt$$
$$= \int_0^1 \sqrt{9(t^2+2)^2}\, dt$$
$$= \int_0^1 3(t^2+2)\, dt$$
$$= \int_0^1 3t^2 + 6\, dt$$
$$= t^3 + 6t \Big|_0^1 = \boxed{7}$$