13.4 Motion in Space

We introduce how derivatives and integrals of vector functions can be used to answer questions about position, velocity and acceleration in 3 dimensions.

If t represents time and $\mathbf{r}(t) = \langle x(t), y(t), z(t)\rangle$ represents the position of a moving object, then we have

$$\mathbf{v}(t) = \mathbf{r}'(t) \quad \text{the velocity vector}$$

$$|\mathbf{v}(t)| = |\mathbf{r}'(t)| \quad \text{the speed function}$$

$$\mathbf{a}(t) = \mathbf{r}''(t) \quad \text{the acceleration vector}$$

Measuring Acceleration
The acceleration can be decomposed into two parts: a tangential component and a normal component. A description of their interpretations is given in the text and in lecture. To compute them, you can use the following:

$$\begin{aligned} a_T &= \frac{\mathbf{r}'(t)\cdot\mathbf{r}''(t)}{|\mathbf{r}'(t)|} && \text{(tangential component)} \\ a_N &= \frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|} && \text{(normal component)} \end{aligned}$$

Modeling Motion
From Newton's Second Law of Motion, we have $\mathbf{F}(t) = m\mathbf{a}(t)$.
That is, the force vector is equal to mass times the acceleration vector. Please several examples in the text and lecture for some uses of this law. We use it to model any motion problem! First, you find all the forces acting on an object which gives you the acceleration. Then you integral or solve a differential equation to find the velocity and position.