MATH 126 – EXAM II Hints and Answers
Version Alpha
Autumn 2009

1. (a) (7 points) HINT: $r'(2) = \langle 4, -2, 6\rangle$ and $r''(2) = \langle 2, 1, 1\rangle$.
   ANSWER: $a_T = \dfrac{12}{\sqrt{56}}$ and $a_N = \dfrac{8\sqrt{3}}{\sqrt{56}}$
   (b) (3 points) ANSWER: $4(x-4) - 2(y+5) + 6(z-10) = 0$ OR $4x - 2y + 6z = 86$ OR $2x - y + 3z = 43$

2. (a) (4 points) HINT: $f_y(x, y) = -e^{-xy}(\sin y + x\cos y)$
   ANSWER: $f_{yx}(x, y) = -e^{-xy}(\cos y - y\sin y - xy\cos y)$
   (b) (4 points) HINT: $f_x(x, y) = -ye^{-xy}\cos y$. So, $f_x(\pi, 0) = 0$ and $f_y(\pi, 0) = -\pi$. The tangent plane is the plane with normal vector $\langle 0, -\pi, -1\rangle$ that contains the point $(\pi, 0, f(\pi, 0)) = (\pi, 0, 1)$.
   ANSWER: $-\pi(y-0) - 1(z-1) = 0$ OR $z = 1 - \pi y$
   (c) (2 points) ANSWER: $f(3.15, 0.001) \approx 1 - 0.001\pi \approx 0.9968584$

3. (a) (8 points) HINT: $g_x(x, y) = x + y - 3$ and $g_y(x, y) = x + y^2 - 3$.
   ANSWER: There is a saddle point at $(3, 0)$ and a local minimum at $(2, 1)$.
   (b) (2 points) HINT: $g(x, 0) = \frac{1}{2}x^2 - 3x$, a quadratic whose graph is a parabola that opens up. Its vertex occurs at $x = 3$.
   ANSWER: $g(3, 0) = -\frac{9}{2}$

4. HINT: You must change the order of integration! With the current order, you have $0 \le x \le \sqrt{\pi/2}$ and $x \le y \le \sqrt{\pi/2}$. This means, the region over which you are integrating is the triangle bounded on the left by the y-axis ($x = 0$), below by the line $y = x$ and above by the line $y = \sqrt{\pi/2}$.

   Then, we have:

$$\int_0^{\sqrt{\pi/2}} \int_x^{\sqrt{\pi/2}} \cos(y^2)\, dy\, dx = \int_0^{\sqrt{\pi/2}} \int_0^{y} \cos(y^2)\, dx\, dy.$$

   ANSWER: $\frac{1}{2}$

5. HINT: Convert to polar:

$$\iint_D \frac{xye^x}{(x^2+y^2)^{3/2}}\, dA = \int_0^{\pi/2} \int_0^3 \cos\theta \sin\theta e^{r\cos\theta}\, dr\, d\theta.$$

   ANSWER: $\dfrac{1}{3}e^3 - \dfrac{4}{3}$