Review Problems for Exam II
MATH 126 – Winter 2010

Here are some problems from old exams that will help you prepare for the midterm. This is not intended to be an exhaustive review of the material. You will be expected to understand how to do all the assigned homework. I suggest you work these problems only after re-working most or all of the homework.

1. Suppose a particle is moving in 3-dimensional space so that its position vector is

$$\overrightarrow{r}(t) = \langle t, t^2, \frac{1}{t} \rangle.$$

   (a) Find the tangential component of the particle's acceleration vector at time $t = 1$.
   (b) Find all values of t at which the particle's velocity vector is orthogonal to the particle's acceleration vector.

2. A particle is moving so that its position is given by the vector function

$$\vec{r}(t) = \langle t^2, t, 5t \rangle$$

   Find the tangent and normal components of the particle's acceleration vector.

3. Let $f(x, y) = xe^y - \ln(x + y)$.
   (a) Sketch the domain of f.
   (b) Find $f_{xy}(x, y)$.

4. Let $f(x, y) = x^2 y + x \sin y - \ln(x - y^2)$.
   (a) Find $f_y(x, y)$.
   (b) Find $f_{xy}(x, y)$.

5. Let $f(x, y) = x^4 y^3 - 3xy^2 + 4x^5 - \frac{6}{y^2} + (e^{x^3 - x})(\ln y)$. Compute f_x, f_{xx}, and f_{xy}.

6. Consider the function

$$f(x, y) = \sqrt{4 + 2x^2 - 3y^2}.$$

   (a) Describe and graph the level set of f of level $c = 2$.
   (b) Find an equation of the tangent plane to the surface $z = f(x, y)$ at the point $(2, 1, 3)$.
   (c) Use the linear approximation to approximate $f(1.9, 1.2)$.

7. Let

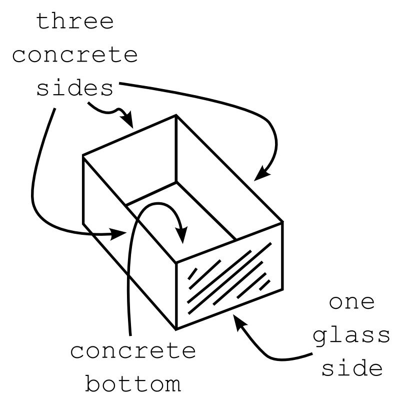
$$f(x, y) = \frac{2x^2 + y^2}{\ln(2x - y)}.$$

   (a) Find and sketch the domain of f.
   (b) Consider the surface $z = f(x, y)$. Find the equation of the tangent plane to the surface at a point (x_0, y_0, z_0) with $x_0 = y_0 = e$.
   (c) Using the linear approximation at (e, e) estimate $f(3, 3)$.

8. Find the local maximum and minimum values and the saddle points of the function

$$f(x, y) = x^3 - 12x - 6y + y^2 + 1.$$

9. You wish to build a large swimming pool in the shape of a parallelpiped. It will essentially be an open-top box made of concrete. One side, however, will be made of glass, so that the pool can be observed from below ground.



Concrete costs \$15 per square meter, and glass costs \$100 per square meter. If the volume of the pool must be 1000 cubic meters, what should the dimensions be to minimize the cost of the pool?

10. Find and classify all the critical points of the function $f(x, y) = 3xy - x^3 - y^3$.

11. Let $D = \{(x, y) \in \mathbb{R}^2 \mid 4 \le x^2 + y^2 \le 4x,\ y \ge 0\}$.

    (a) Draw a careful picture for the domain D.

    (b) Compute the area of D.

12. (a) Find the area of the region bounded by the x-axis and the cardiod $r = 1 + \cos\theta$ from $\theta = 0$ to $\theta = \pi$.

    (b) Let R be the region in the first quadrant of the xy-plane that lies inside the cardioid $r = 1 + \cos\theta$ and outside the circle $r = 1$. Find the volume of the solid that lies above R and below the plane $z = y$.

13. Evaluate the integral $\displaystyle\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y}\, dy\, dx.$

14. A lamina occupies the region in the xy-plane bounded by the lines $x = 1$, $x = 2$, $y = ax$, and $y = 2ax$ for some positive number a. The lamina has density function $\rho(x, y) = \frac{1}{x} + \frac{1}{y^2}$. Find the value of a that minimizes the mass of the lamina.

15. A sheet of material covering the first quadrant ($x \ge 0$, $y \ge 0$) has a surface density (mass per area) given by
$$\rho(x, y) = 3e^{-2x-3y}.$$
What is the total mass of that part of the sheet enclosed by the triangular area whose vertices are located at $(1, 0)$, $(0, 0)$, $(0, 1)$?

16. Evaluate
$$I = \int_1^{\ln 8} \int_0^{\ln y} e^{x+y}\, dxdy.$$

17. Consider the region R in the xy-plane that sits above the x-axis and is bounded by $x^2 + y^2 = 4$ and $x^2 + y^2 = 1$. Compute the volume of the solid that is bounded below by R and above by the function $f(x, y) = e^{-(x^2+y^2)}$.