Worksheet 3: Surfaces, Parametric Facts, and Polar Facts

This worksheet reviews and summarizes several topics that you are learning right now (or are about to learn). The front should be mostly review and the back introduces polar coordinates. Use this time to discuss these new topics in groups and to clear up confusion with your TA.

1. *Surfaces:* Consider the surface given by $x + y^2 - z^2 = 4$.
   - If $x = k$ is fixed, identify the trace.
   - If $y = k$ is fixed, identify the trace.
   - If $z = k$ is fixed, identify the trace.
   - What is the name of this shape?

2. *Basic Parametric:* Consider the curve given by $x = t$, $y = t\sin(\pi t)$, $z = t\cos(\pi t)$. Eliminate the parameter in order to get a surface over which this motion is occurring (you should know the name of this surface).

3. *Basic Parametric:* Two objects are traveling through space with vector functions given by $\mathbf{r}_1(t) = \langle t, 5t, t^2 \rangle$ and $\mathbf{r}_2(t) = \langle 5 - t, 7t + 1, t^3 + 1 \rangle$.
   - Find all points at which their **paths** intersect. (Hint: Be careful with parameters).
   - Do the object every **collide**? (If so, find the time when they collide. If not, explain why.)

4. *Parametric Calculus:* For the curve given by $\mathbf{r}(t) = \langle t^2 + 1, t^3, 1 - 5t \rangle$.
   - Find $r'(t)$.
   - Find $r'(1)$ and $T(1)$.
   - Find parametric equations for the tangent line to the curve at $t = 1$.

An introduction to Polar Coordinates:
Assume you are standing at the origin facing the positive x-axis. If you rotate counterclockwise by an angle of θ and walk in the new direction you are facing by r units, then you have reach a point using the polar coordinate method. Some conventions:

- Positive θ means rotate counterclockwise and negative θ means rotate clockwise.
- Postiive r means walk forward and negative r means walk backward.

You should already know from other things we have done this term that

$$x = r\cos(\theta)\ ,\ y = r\sin(\theta)\ ,\ x^2 + y^2 = r^2\ ,\ \tan(\theta) = \frac{y}{x}.$$

1. Sketch the region in the xy-plane given by all points (x, y) such that $1 \leq x \leq 4$ and $-5 \leq y \leq -2$ (this a region easily described in Cartesian coordinates)

2. Sketch the region in the xy-plane given by all polar points (r, θ) such that $-\frac{\pi}{2} \leq \theta \leq \pi$ and $2 \leq r \leq 5$ (this is a region much more easily described in Polar coordinates).

3. Let's plot a couple polar curves. I want you to just make a table (including $\theta = 0, \pi/6, \pi/4, \ldots, 2\pi$), plot the points, and connect the dots.

   - $r = 1 + \cos(\theta)$ (this is called a Cardiod, it sort of looks like a heart)
   - $r = \theta$

4. In 10.2 you learned that if x and y are given in terms of a parameter, then you can find the slope directly from the parameter by using $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$. You can use this to find slopes directly from polar curves. The general presentation would be: Suppose $r = f(\theta)$. From our known connections $x = f(\theta)\cos(\theta)$ and $y = f(\theta)\sin(\theta)$. Note, you can find $dx/d\theta$ using the product rule (same for $dy/d\theta$). Thus, to find the slope you can use

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{f'(\theta)\sin(\theta) + f(\theta)\cos(\theta)}{f'(\theta)\cos(\theta) - f(\theta)\sin(\theta)}$$

   For the cardiod $r = 1 + \cos(\theta)$, use the fact above to find the formula for dy/dx. What is the slope when $\theta = \pi/2$? How about when $\theta = \pi/4$?