## Exam 1

April 25, 2013

Name: $\qquad$
Section: $\qquad$

Student ID Number:

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- There are 4 pages of questions. Make sure your exam contains all these questions.
- You are allowed to use a scientific calculator (no graphing calculators and no calculators that have calculus capabilities) and one hand-written 8.5 by 11 inch page of notes.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit. Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded. Give exact answers wherever possible.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Cheating will not be tolerated. Keep your eyes on your exam! Anyone found engaging in academic misconduct will receive a zero on the exam.
- You have 50 minutes to complete the exam. Budget your time wisely.

SPEND NO MORE THAN 10 MINUTES PER PAGE!

1. (a) (6 pts) Find the equation of the plane containing the point $(1,-1,2)$ and line $L:\left\{\begin{array}{l}x=-2 t ; \\ y=1 ; \\ z=2+t\end{array}\right.$
(b) (10 pts) Consider the line, $L$ that goes through the two points $(5,0,10)$ and $(4,1,8)$ and consider the plane, $P$, given by $2 x+3 y-z=2$.
i. Find the $(x, y, z)$ coordinates at which the line, $L$, intersects the plane, $P$.
ii. The angle of entry for the intersection of a line and a plane is defined to be the acute angle between the line and the normal direction for the plane. Find the angle of entry for the intersection of the line, $L$, and the plane, $P$. (Give in degrees rounded to two places after the decimal).
2. (11 points)
(a) (6 pts) Assume $\mathbf{a}$ and $\mathbf{b}$ are nonzero three-dimensional vectors that are not parallel and are not orthogonal.
In each case below, determine if the two vectors are always are orthogonal, always are parallel, always are neither parallel or perpendicular, or it depends on the vectors (meaning depending on the vectors it is possible they could be perpendicular or parallel or neither).
Circle one for each (no work is necessary):

| i. $\mathbf{a} \times \mathbf{b}$ and $3 \mathbf{a}$. | orthogonal | parallel | neither | depends |
| :--- | :--- | :--- | :--- | :--- |
| ii. $\mathbf{p r o j}_{\mathbf{a}}(\mathbf{b})$ and $\mathbf{b}$. | orthogonal | parallel | neither | depends |
| iii. $\frac{1}{\|\mathbf{a}\|} \mathbf{a}$ and $(\mathbf{a} \cdot \mathbf{b}) \mathbf{a}$. | orthogonal | parallel | neither | depends |
| iv. $\mathbf{b}-\operatorname{proj}_{\mathbf{a}}(\mathbf{b})$ and $\mathbf{a}$. | orthogonal | parallel | neither | depends |
| v. $\mathbf{a}+\mathbf{b}$ and $\mathbf{a}-\mathbf{b}$. | orthogonal | parallel | neither | depends |
| vi. $\mathbf{a}-\mathbf{b}$ and $\mathbf{b}-\mathbf{a}$. | orthogonal | parallel | neither | depends |

(b) (5 pts) Consider the parametric curve given by $x=t^{2}-2 t, y=t^{3}-4 t$. Find all times $t$ at which the tangent line to the curve is orthogonal to the vector $\langle 2,-1\rangle$.
3. (a) ( 6 pts ) In the blanks provided to the left of each graph, put the number of the polar equation that matches the graph in the $x y$-plane (two graphs will not be labeled).

1. $r=\sqrt{\theta}$
2. $r=1-2 \cos (\theta)$
3. $r=1+\sin (2 \theta)$
4. $r=9 \cos (\theta)$

(b) (5 pts) Find the $(x, y)$ coordinates of all points on the curve $r=1-2 \cos (\theta)$ that intersect the line $y=x$.
5. (12 pts) Dr. Loveless has motion sickness. You trick him into getting on a roller coaster that follows the path given by the vector function: $\mathbf{r}(u)=\langle 20 \sin (u), 24 u, 20 \cos (u)+40\rangle$.
Assume $u=0$ corresponds to the start of the ride and that the ride starts at rest. All distances are in feet.
(a) When the ride gets to the point $(x, y, z)=(10 \sqrt{3}, 8 \pi, 50)$, Dr. Loveless' calculator falls out of his pocket. Assume the calculator follows the path of the tangent line (there happens to be no gravity). If the $x y$-plane is the ground, at what location $(x, y, z)$ does the calculator land on the ground?
(b) If the magnitude of acceleration of the roller coaster is always a constant $4 \mathrm{ft} / \mathrm{s}^{2}$, how long did it take for Dr. Loveless to get to the point $(x, y, z)=(10 \sqrt{3}, 8 \pi, 50)$ on the curve?
(Hint: Start by finding the distance traveled).
