Polar Coordinate Overview

The polar worksheet may be slightly ahead of the material in lecture. However, it still is a useful tool to give you an introduction to the concepts pertaining to polar coordinates. Please read through this supplement before going to quiz section for the polar worksheet on Thursday.

Introduction

There is more than one way to describe a location in a coordinate system. You are all familiar with the Cartesian coordinate method.

Cartesian method: (x, y)

1. Stand on the origin.
2. First, walk x units on the x-axis.
3. Then, walk y units parallel to the y-axis.

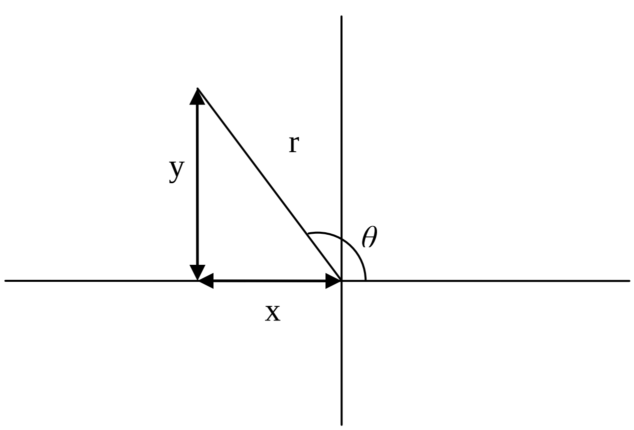
This is kind of like describing how to get somewhere by driving along streets.

However, in some scenarios it is more convenient to give the location in terms of an angle and a radius. For example, imagine you are firing a cannon (you need to know where to aim it and how powerful to shoot it). We call this the Polar coordinate method.

Polar coordinates: (r, θ)

1. Stand on the origin facing the positive x-axis.
2. Rotate counterclockwise by the angle θ.
3. Walk (or fire your cannon) a distance r in the direction you are facing.

Note: If r is negative then you walk backward from the direction you are facing. Here is an illustration of this situation. Note that these methods both get you to the same location. We will find the polar coordinates and polar equations can greatly simplify certain problems, especially those involving circles and ellipses.



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You can go back and forth between Cartesian and polar coordinates, by using the connections given in the text. (These all come from basic properties of Sine, Cosine, and Tangent).

Graphing

When you first learned to graph equations such as $y = x^2$ and $y = e^x$, you had to plot points to get an idea of what the graph looked like. That is, you had to make a table by selecting values of x and computing values of y. And then you plotted the (x, y) coordinates using the Cartesian coordinate method. Note that as x gets bigger the graph moves to the right as we are accustomed.

Since we are new to polar coordinates, you will have to use the same idea to plot polar equations. You have to make a table by selecting values of θ and computing values of r. And then you plot (r, θ) using the Polar coordinate method. Here as θ increases the graph moves in a counterclockwise direction, so the graph are often "spiraling" in some way.

There is one other plotting option for Polar coordinates. You can first change the variables from r and θ to x and y. Then use what you know about plotting in Cartesian coordinates. In order to change the variables you will need to use the identities from and you will have to do some algebra. You will practice this idea in the worksheet and homework.

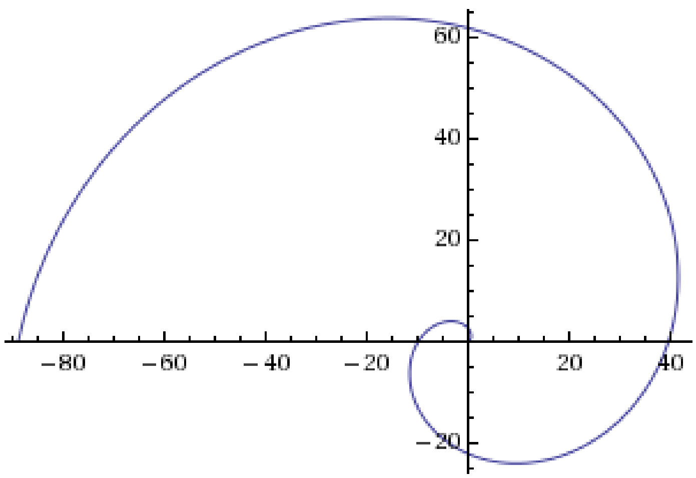
Below are two examples of graphing by plotting points.

Example 1:

$$r = \theta^2$$

To the right you can see that a few calculated points. From the table we see that as θ spirals around r gets bigger and bigger. So $r = \theta^2$ is spiraling out. Below is the picture as far as we have plotted.

θ	r
0	0
$\pi/2$	$(\pi/2)^2 \approx 2.47$
π	$(\pi)^2 \approx 9.87$
$3\pi/2$	$(3\pi/2)^2 \approx 22.21$
2π	$(2\pi)^2 \approx 39.48$
$5\pi/2$	$(5\pi/2)^2 \approx 61.69$
3π	$(3\pi)^2 \approx 88.83$



If r is any strictly increasing (or strictly decreasing) function of θ, then the graph will have this spiraling in or spiraling out behavior. In many situations where we use polar coordinates, the function for r in terms of θ involves trig functions (which oscillate and don't strictly increase or decrease).

Example 2:

$r = \cos(3\theta)$

Notice that we pick points to plot so that the input to cosine (that is, 3θ, comes out to be the multiples of $\pi/4$, which we know for cosine).

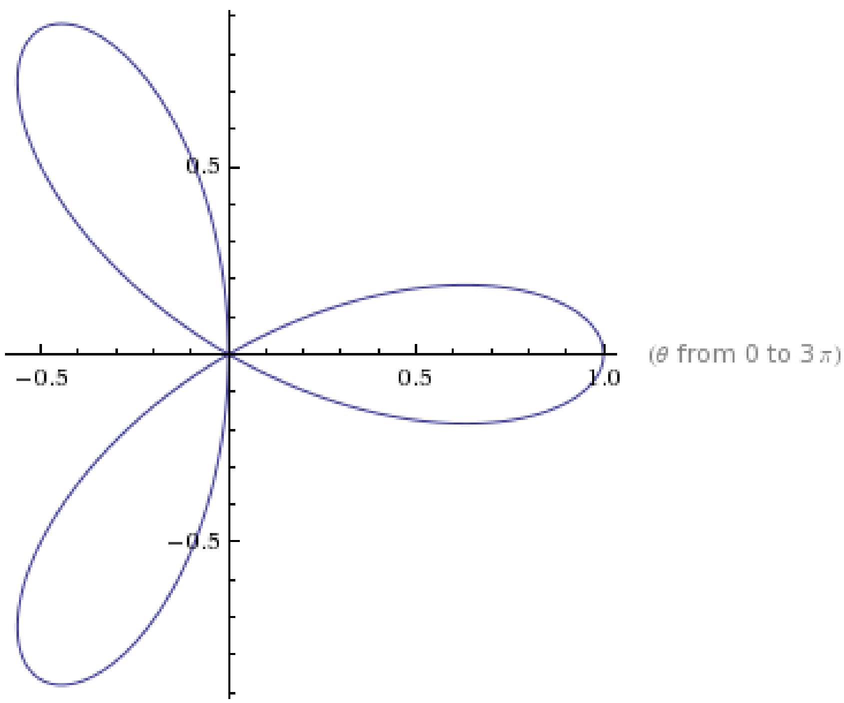
So as θ spirals from 0 to $\pi/6$, r decreases to the origin.

As θ spirals from $\pi/6$ to $\pi/2$, r is negative (so in the third quadrant) and gets to maximum of 1 before returning to the origin.

As θ spirals from $\pi/2$ to $5\pi/6$, r is positive (so in the second quadrant) and gets to a maximum of 1 before returning to the origin.

As θ spirals from $5\pi/6$ to π, r is negative (so in the fourth quadrant) and ends up back at the location we started out. Then it all repeats. Here is the graph:

θ	r
0	1
$\pi/12$	$\sqrt{2}/2$
$\pi/6$	0
$\pi/4$	$-\sqrt{2}/2$
$\pi/3$	-1
$5\pi/12$	$-\sqrt{2}/2$
$\pi/2$	0
$7\pi/12$	$\sqrt{2}/2$
$4\pi/3$	1
$3\pi/4$	$\sqrt{2}/2$
$5\pi/6$	0
$11\pi/12$	$-\sqrt{2}/2$
π	-1



Now Attempt The Worksheet on Polar Coordinates on Thursday