Math 126 Exam 2 Quick Review

**13.3**: Measurement on 3D Curves

1. Arc Len. $= \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}\, dt$.

2. $\kappa(t) = \frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$ (in 2D, $\kappa(x) = \frac{|f''(x)|}{(1+(f'(x))^2)^{3/2}}$)

3. $\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$, $\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|}$, $\mathbf{B} = \mathbf{T}\times\mathbf{N}$.
   Conceptual reminder: $\mathbf{r}'(t_0)$, $\mathbf{r}''(t_0)$, $\mathbf{T}(t_0)$, and $\mathbf{N}(t_0)$ are all on the same plane (the osculating plane).

4. Tangent Line:
   Through $\mathbf{r}(t_0)$ in direction of $\mathbf{r}'(t_0)$.

5. Normal Plane:
   Through $(x_0, y_0, z_0)$ with normal in direction of $\mathbf{r}'(t_0)$.

6. Osculating Plane:
   Through $(x_0, y_0, z_0)$ with normal in direction of $\mathbf{B}(t_0)$.

**13.4**: Velocity and Acceleration

1. If $t$ is time, then
   $\mathbf{r}(t)$ = position
   $\mathbf{v}(t) = \mathbf{r}'(t)$ is velocity, $|\mathbf{v}(t)|$ is speed, and
   $\mathbf{a}(t) = \mathbf{r}''(t)$ is acceleration.
   Be able to go from position to acceleration and acceleration to position. (Be careful with constants of integration).

2. $a_T = \frac{\mathbf{r}'(t)\cdot\mathbf{r}''(t)}{|\mathbf{r}'(t)|}$, $a_N = \frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|}$

**14.1, 14.3, 14.4**: Multivariable functions, Partials

1. Sketch a domain and sketch level curves.
2. Compute partial derivatives and understand what they represent.
   $f_x(x_0, y_0)$ = 'slope in $x$-direction'
   $f_y(x_0, y_0)$ = 'slope in $y$-direction'
3. Find a tangent plane, a linearization, and the total differential:

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0).$$

$$L(x, y) = z_0 + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0).$$

$$dz = f_x(x_0, y_0)dx + f_y(x_0, y_0)dy$$

**14.7**: Critical points and max/min

1. Find critical points:
   Set $f_x(x, y) = 0$ and $f_y(x, y) = 0$,
   then COMBINE and solve (check your answers).
2. Classify critical points (second derivative test).
   $D = f_{xx}f_{yy} - f_{xy}^2$.
   If $D > 0$ and $f_{xx} > 0$, then local min.
   If $D > 0$ and $f_{xx} < 0$, then local max.
   If $D < 0$, then saddle point.
3. Find the absolute max/min over a region.
   (a) Find critical points.
   (b) Find the critical numbers on each boundary.
   (c) Evaluate the original function at all critical points inside and critical numbers on the boundaries and all corners.

**15.1-15.4**: Double Integrals.

1. Break up a rectanglular domain into rows and columns and approximate the volume with rectangular boxes.

2. $\displaystyle\iint_D f(x,y)\,dA =$ signed volume 'above' the region $D$ in the $xy$-plane and 'below' $f(x,y)$.

3. Other applications:

   $\displaystyle\iint_D 1\,dA =$ area of $D$.

   $\displaystyle\frac{1}{\text{Area of D}}\iint_D f(x,y)\,dA =$ average value of $f(x,y)$.

4. To set up a double integral from a description:

   (a) Solving for integrand(s) (*i.e.* get $z = f(x,y)$).

   (b) Draw given $xy$-equations in the $xy$-plane (label all intersection points).

   (c) Draw any $xy$-equations that occur from intersections of surfcaces (*i.e.* $z = f(x,y)$ with $z = 0$ or the intersection of two given surfaces).

   (d) Set up the double integral(s) using the region for $D$.

5. Options for set up:

$$\iint_D f(x,y)\,dA = \int_a^b \int_{g(x)}^{h(x)} f(x,y)\,dydx$$

$$\iint_D f(x,y)\,dA = \int_c^d \int_{p(y)}^{q(y)} f(x,y)\,dxdy$$

$$\iint_D f(x,y)\,dA = \int_\alpha^\beta \int_{w(\theta)}^{v(\theta)} f(r\cos(\theta), r\sin(\theta))\,rdrd\theta$$

6. Be able to DRAW the region $D$ if given a double integral that is already set up and then reverse the order.