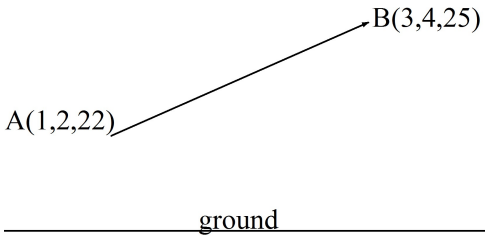
Worksheet 2c: Correctly Parameterizing a Line

Story: You are watching Gary Potter on his magical flying broom. You impose a coordinate system so that the ground is the xy-plane. At precisely noon, the back of the broom is at the point P(1,2,22) and the front of the broom is at the point Q(3,4,25), so Gary is flying upward. Below is a picture of the view from the side.



1. Find any parametric equation for the line through the broom (make your direction vector point in the direction Gary is flying).

   - For the parameterization you just gave, where is the front of the broom when your parameter is $t = 0$ and when it is $t = 1$.
   - Compute the distance traveled during that timespan from $t = 0$ to $t = 1$. Note that this is your constant speed (the distance you travel in $t = 1$ second).

   Go back and adjust your parameterization so that when you plug in $t = 0$ you get $(x(0), y(0), z(0)) = (3, 4, 25)$ (so the equations are describing the motion of the point on the front of the broom).

2. Give another parameteric equation for the same motion but use a unit vector for the direction vector.

   - For this parameterization where is the front of the broom when your parameter is $t = 0$ and when it is $t = 1$.
   - Compute the distance traveled during that timespan from $t = 0$ to $t = 1$. Note that this is your constant speed (the distance you travel in $t = 1$ second).

   You should see that in $t = 1$ second, you travel exactly $d = 1$ foot. We call this parameterizing in terms of distance (or arc length) since t is the same as distance d. For whatever you used for the parameter in this part (probably t), erase t and replace it with d. Now you have a parameterization in terms of distance traveled.

3. Assume Gary is traveling at a constant speed of 24 feet/second. Give a correct parameterization. (Hint: What do you think the formula is for distance? Now replace d in the last part by this formula. You can check your work, in one second Gary should go 24 feet).

   - In this scenario, how long will it take Gary to get to 500 feet?

4. Assume Gary is at rest at noon, then accelerates at a constant rate of 3 feet/sec^2. Find the parameterization of the motion in terms of time, t, seconds after noon. (Hint: Start with $a(t) = 3$, then use calculus to find distance $d(t)$. Then replace d from the earlier part).

   - In this scenario, how long will it take Gary to get to 500 feet?