1. (16 points)

(a) Find the **equation of the plane containing the point** $(1,-1,2)$ and the line, $L: \begin{cases} x = -2t; \\ y = 1; \\ z = 2+t. \end{cases}$

$\langle -2,0,1\rangle$ is parallel to the plane

$\langle 0,1,2\rangle - \langle 1,-1,2\rangle = \langle -1,2,0\rangle$ also is parallel to plane.

NORMAL: $\langle -2,0,1\rangle \times \langle -1,2,0\rangle = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 0 & 1 \\ -1 & 2 & 0 \end{vmatrix} = \langle -2,-1,-4\rangle$

PLANE: $\boxed{\begin{array}{c} -2(x-1)-(y+1)-4(z-2)=0 \\ 2(x-1)+(y+1)+4(z-2)=0 \\ 2x+y+4z=9 \end{array}}$

(b) Consider the line, L, that goes through the two points $(5,0,10)$ and $(4,1,8)$ and consider the plane, P, given by $2x+3y-z=2$.

i. Find the (x,y,z) coordinates at which the line intersects the plane.

DIRECTION: $\langle -1,1,-2\rangle$

LINE: $x = 5-t$
$y = t$
$z = 10-2t$

INTERSECT: $2(5-t)+3t-(10-2t)=2$

$\Rightarrow$ $10-2t+3t-10+2t=2$
$3t=2$
$t = 2/3$ $\Rightarrow$

$\boxed{\begin{array}{l} x = 5 - 2/3 = \frac{13}{3} \\ y = 2/3 \\ z = 10-2(2/3) = \frac{26}{3} \end{array}}$

ii. The *angle of entry* for the intersection you just found is defined to be the acute angle between the line and normal direction for the plane. Find the angle of entry for the intersection of the ~~original~~ line, ~~L~~, and this plane. (Give in degrees rounded to two digits)

ACUTE ANGLE BETWEEN $\langle -1,1,-2\rangle$ and $\langle 2,3,-1\rangle$

$\langle -1,1,-2\rangle \cdot \langle 2,3,-1\rangle = \sqrt{1+1+4}\sqrt{4+9+1}\cos\theta$

$-2+3+2 = \sqrt{6}\sqrt{14}\cos\theta$

$\theta = \cos^{-1}\left(\frac{3}{\sqrt{6}\sqrt{14}}\right) \approx 70.8939465$

$\approx \boxed{70.89°}$

acute ✓

ASIDE

1.2373 radians

2. (11 points)

(a) (6 pts) Assume **a** and **b** are nonzero three-dimensional vectors that are not parallel and are not orthogonal.

In each case below, determine if the two vectors are *always are orthogonal*, *always are parallel*, can *never be either parallel or perpendicular*, or *it depends on the vectors* (meaning depending on the vectors it is possible they could be perpendicular or parallel or neither). **(Circle one for each)**:

i. $\underbrace{\mathbf{a} \times \mathbf{b}}_{\text{orthogonal to } \vec{a} \text{ and } \vec{b}}$ and $\underbrace{\overset{1}{\mathbf{a}}}_{\text{same direction as } \vec{a}}$.	orthogonal	parallel	neither	depends
ii. $\mathbf{proj_a}(\mathbf{b})$ and $\mathbf{b}$.	orthogonal	parallel	neither	depends
iii. $\frac{1}{\lvert\mathbf{a}\rvert}\mathbf{a}$ and $(\mathbf{a} \cdot \mathbf{b})\mathbf{a}$.	orthogonal	parallel	neither	depends
iv. $\mathbf{b} - \mathbf{proj_a}(\mathbf{b})$ and $\mathbf{a}$.	orthogonal	parallel	neither	depends
v. $\mathbf{a} + \mathbf{b}$ and $\mathbf{a} - \mathbf{b}$.	orthogonal	parallel	neither	depends
vi. $\mathbf{a} - \mathbf{b}$ and $\mathbf{b} - \mathbf{a}$.	orthogonal	parallel	neither	depends

(b) (5 pts) Consider the parametric curve given by $x = t^2 - 2t$, $y = t^3 - 4t$. Find all times t at which the tangent line to the curve is orthogonal to the vector $\langle 2, -1 \rangle$.

TWO WAYS

① 

$\langle 2,-1\rangle$ orthogonal $\Rightarrow$ tangent has slope $= 2$

OR

② WANT $\left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle$ to be orthogonal to $\langle 2,-1\rangle$.

$$\langle 2t-2, 3t^2-4\rangle \cdot \langle 2,-1\rangle = 0$$

$$\Rightarrow \quad 4t-4-(3t^2-4) = 0$$

$$4t-3t^2 = 0$$

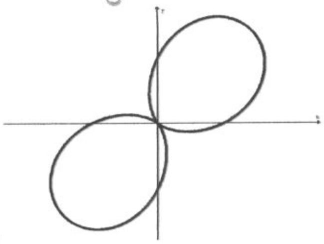
$$t(4-3t) = 0$$

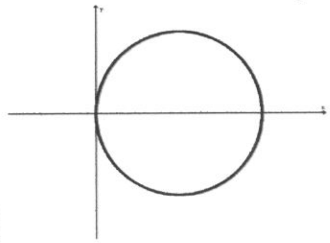
$$\boxed{t=0} \quad \text{or} \quad \boxed{t = 4/3}$$

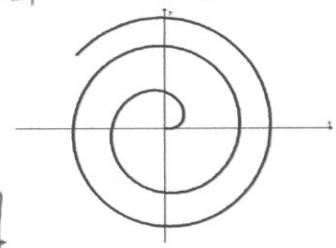
PLOT POINTS

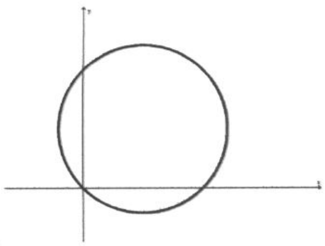
3. (a) (6 pts) In the blanks provided to the left of each graph, put the number of the polar equation that matches the graph in the xy-plane (two graphs will not be labeled).

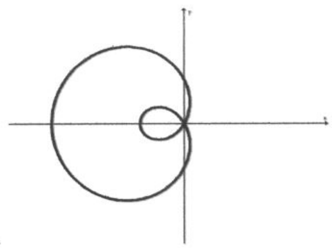
1. $r = \sqrt{\theta}$	2. $r = 1 - 2\cos(\theta)$	3. $r = 1 + \sin(2\theta)$	4. $r = 9\cos(\theta)$
r increasing, spiraling out ✓	$\theta = 0 \Rightarrow r = -1$ $\theta = \pi \Rightarrow r = 3$ ✓	$\theta = 0 \Rightarrow r = 1$ $\theta = \pi/2 \Rightarrow r = 1$ ✓ $\theta = \pi \Rightarrow r = 1$	$\theta = 0 \Rightarrow r = 9$ $\theta = \pi/2 \Rightarrow r = 0$ ✓ $\theta = \pi \Rightarrow r = -9$

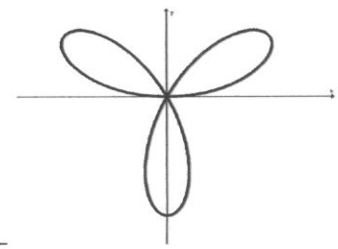
$\boxed{3}$ 

$\boxed{4}$ 

$\boxed{1}$ 

____ 

$\boxed{2}$ 

____ 

(b) (5 pts) Find the (x, y) coordinates of all points on the curve $r = 1 - 2\cos(\theta)$ that intersect the line $y = x$.

Either convert all to cartesian or all to polar. → easier

$$y = x \Rightarrow \theta = \frac{\pi}{4} \quad \text{or} \quad \theta = \frac{3\pi}{4} \quad \text{or} \quad r = 0$$

$$\theta = \pi/4 \Rightarrow r = 1 - 2\frac{\sqrt{2}}{2} = 1 - \sqrt{2} \Rightarrow \begin{array}{l} x = (1-\sqrt{2})\cos(\pi/4) = \frac{\sqrt{2}}{2} - 1 \\ y = (1-\sqrt{2})\sin(\pi/4) = \frac{\sqrt{2}}{2} - 1 \end{array}$$

$$\theta = 3\pi/4 \Rightarrow r = 1 + 2\frac{\sqrt{2}}{2} = 1 + \sqrt{2} \Rightarrow \begin{array}{l} x = (1+\sqrt{2})\cdot -\frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2} - 1 \\ y = (1+\sqrt{2})\cdot -\frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2} - 1 \end{array}$$

$$r = 0 \Rightarrow \begin{array}{l} x = 0 \\ y = 0 \end{array}$$

$$\boxed{\left(\frac{\sqrt{2}}{2} - 1, \frac{\sqrt{2}}{2} - 1\right), \left(-\frac{\sqrt{2}}{2} - 1, -\frac{\sqrt{2}}{2} - 1\right), (0, 0)}$$

4. (12 pts) Dr. Loveless has motion sickness. You trick him into getting on a roller coaster that follows the path given by the vector function: $\mathbf{r}(u) = \langle 20\sin(u), 24u, 20\cos(u) + 40\rangle$.
Assume $u = 0$ corresponds to the start of the ride and that the ride starts at rest. (Distances are all in feet)

(a) When the ride gets to the point $(x, y, z) = (10\sqrt{3}, 8\pi,$ [redacted] $)$ 50, Dr. Loveless' calculator falls out of his pocket. Assume the calculator follows the path of the tangent line (there happens to be no gravity). If the xy-plane is the ground, at what location (x, y, z) does the calculator land on the ground?

$20\sin(u) = 10\sqrt{3},\ 24u = 8\pi,\ 20\cos(u) + 40 = 50 \Rightarrow u = \pi/3$

$\vec{r}\,'(u) = \langle 20\cos(u), 24, -20\sin(u)\rangle$

$\vec{r}\,'(\pi/3) = \langle 10, 24, -10\sqrt{3}\rangle$

TANGENT LINE: $x = 10\sqrt{3} + 10t$
$y = 8\pi + 24t$
$z = 50 - 10\sqrt{3}t$

$z = 0 \Rightarrow t = \frac{50}{10\sqrt{3}} = \frac{5}{\sqrt{3}} = \frac{5}{3}\sqrt{3}$

$$\boxed{\begin{aligned}(x,y,z) &= \left(10\sqrt{3} + \tfrac{50}{3}\sqrt{3},\ 8\pi + 24\tfrac{5}{3}\sqrt{3},\ 0\right)\\ &= \left(\tfrac{80}{3}\sqrt{3},\ 8\pi + 40\sqrt{3},\ 0\right)\end{aligned}}$$

(b) If the magnitude of acceleration of the roller coaster is always a constant 4 ft/s^2, then how long did it take for Dr. Loveless to get to the point $(x, y, z) = (10\sqrt{3}, 8\pi,$ [redacted] $)$ 50 on the curve? (Hint: Start by finding the ~~arc length, i.e.~~ distance traveled).

$$\text{DISTANCE} = \int_0^{\pi/3} \sqrt{(20\cos(u))^2 + 24^2 + (-20\sin(u))^2}\, du$$

$$= \int_0^{\pi/3} \sqrt{20^2 + 24^2}\, du$$

$$= \sqrt{976}\, u \Big|_0^{\pi/3} = \sqrt{976}\, \pi/3 \text{ feet} = 32.7154 \text{ ft}$$

$a(t) = 4$ ft/sec^2 $\Rightarrow$ speed $= v(t) = 4t + C$ $\quad C = 0$ starts at rest

$\Rightarrow$ DISTANCE $= s(t) = 2t^2 + D$ $\quad u = 0 \Rightarrow \text{DIST} = 0$

$2t^2 = \sqrt{976}\, \pi/3$

$t^2 = \frac{\sqrt{976}}{6}\pi$

$$\boxed{t = \sqrt{\frac{\sqrt{976}}{6}\pi} \simeq 4.04447177 \quad 4.04 \text{ seconds}}$$