12.6: Basic 3D Surfaces

Goal: Learn the names of 7 basic 3D surfaces.

Cylinders: If one variable is absent in an equation for a surface, then the graph is really a 2D curve extended into 3D. In these cases we say the shape is a "BLAH" cylinders, where BLAH is the name of the 2D shape. Examples:

1. $x^2 + y^2 = 1$ in $\mathbf{R}^3$ is a circular cylinder (around the z-axis).
2. $z = \cos(x)$ in $\mathbf{R}^3$ is a cosine wave cylinder.

Quadric Surfaces:
These are polynomial equations that involve x, y, and z raised to first and second powers.

Before we can study quadric surfaces, we must know the names of 3 basic 2D curves:

Parabolas: $y = ax^2 + b$.
Ellipses/Circles: $ax^2 + by^2 = c$ (a, b, c all positive)
Hyperbolas: $ax^2 - by^2 = c$ (a, b are positive)

Traces:
Next we need to understand the method of **traces**. A trace of a 3-dimensional curve is a 2-dimensional curve where one of the variables if fixed. If you draw and label several traces, then you get a *contour map* (also called an *elevation map* or a graph of *level curves*). As I talk about surfaces this quarter, we will often talk about traces.

Assume all constant labels are positive:

Equation	Traces (x/y/z)
$az = bx^2 + cy^2$	par./par./ell.
$az = bx^2 - cy^2$	par./par./hyp.
$ax^2 + by^2 + cz^2 = d$	ell./ell./ell.
$ax^2 + by^2 - cz^2 = d$	hyp./hyp./ellipse
$ax^2 + by^2 - cz^2 = 0$	line-hyp./line-hyp./ell.
$ax^2 + by^2 - cz^2 = -d$	hyp./hyp./nothing-ell.

Equation	Name
$az = bx^2 + cy^2$	**Elliptic Paraboloid**
$az = bx^2 - cy^2$	**Hyperbolic Paraboloid**
$ax^2 + by^2 + cz^2 = d$	**Ellipsoid/Sphere**
$ax^2 + by^2 - cz^2 = d$	**Hyperboloid of One Sheet**
$ax^2 + by^2 - cz^2 = 0$	**Cone**
$ax^2 + by^2 - cz^2 = -d$	**Hyperboloid of Two Shts.**