1. (a) (6 pts) Find parametric equations for the line of intersection of the planes $2x - y + 3z + 4 = 0$ and $-x + y - z = 0$.

$\vec{r} = \vec{r}_0 + t\vec{v}$

ONE METHOD: FIND TWO POINTS OF INTERSECTION

① $2x - y + 3z + 4 = 0$

② $-x + y - z = 0$

$x = 0 \Rightarrow$ ① $-y + 3z + 4 = 0$ ② $y - z = 0$

① + ② $\Rightarrow 2z + 4 = 0 \Rightarrow z = -2$ so $y = z = -2$ $P(0, -2, -2)$

$y = 0 \Rightarrow$ ① $2x + 3z + 4 = 0$ ② $-x - z = 0$

① + 2② $\Rightarrow z + 4 = 0 \Rightarrow z = -4$, $x = -z = 4$ $Q(4, 0, -4)$

$\vec{r}_0 = \langle 0, -2, -2 \rangle$

$\vec{v} = \vec{PQ} = \langle 4, 2, -2 \rangle$

$$\boxed{\begin{aligned} x &= 4t \\ y &= -2 + 2t \\ z &= -2 - 2t \end{aligned}}$$

MANY OTHER ANSWERS, BUT ALL MUST HAVE

① DIRECTION PARALLEL TO $\langle 4, 2, -2 \rangle$

② POINTS ON LINE MUST SATISFY

$$\frac{x}{4} = \frac{y+2}{2} = \frac{z+2}{-2}$$

(b) (6 pts) Find the equation of the plane that goes through the two points $P(2, -1, 0)$ and $Q(4, 0, 3)$ and is parallel to the line $x = 3t$, $y = 1 - t$, $z = 4 + t$.

$\vec{n} \cdot (\vec{r} - \vec{r}_0) = 0$

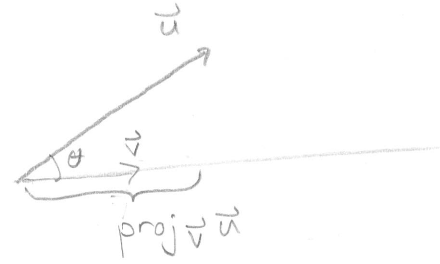
$\vec{r}_0 = \langle 2, -1, 0 \rangle$ (or $\langle 4, 0, 3 \rangle$ or many others)

$$\vec{n} = \vec{PQ} \times \langle 3, -1, 1 \rangle = \langle 2, 1, 3 \rangle \times \langle 3, -1, 1 \rangle = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 3 \\ 3 & -1 & 1 \end{vmatrix}$$

$$= (1 - -3)\vec{i} - (2 - 9)\vec{j} + (-2 - 3)\vec{k} = \langle 4, 7, -5 \rangle$$

$$\boxed{\begin{aligned} &4(x - 2) + 7(y + 1) - 5z = 0 \\ &4x + 7y - 5z - 1 = 0 \end{aligned}}$$

2. Consider the vectors $\mathbf{u} = \langle 3, -2, 5\rangle$, $\mathbf{v} = \langle 2, -1, 0\rangle$.



(a) (4 pts) Find the vector obtained by projecting $\mathbf{u}$ onto $\mathbf{v}$.

DESIRED LENGTH $= |\vec{u}| \cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{|\vec{v}|}$

IN DIRECTION OF THE UNIT VECTOR $\dfrac{1}{|\vec{v}|}\vec{v}$

THUS,

$$\text{proj}_{\vec{v}}(\vec{u}) = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}\vec{v} = \frac{6+2+0}{4+1+0}\langle 2,-1,0\rangle = \boxed{\frac{8}{5}\langle 2,-1,0\rangle = \left\langle \frac{16}{5}, -\frac{8}{5}, 0\right\rangle}$$

(b) (4 pts) Find the area of the triangle with corners $(0, 0, 0)$, $(3, -2, 5)$ and $(2, -1, 0)$.

$$\vec{u}\times\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -2 & 5 \\ 2 & -1 & 0 \end{vmatrix} = (0--5)\vec{i} - (0-10)\vec{j} + (-3--4)\vec{k}$$
$$= \langle 5, 10, 1\rangle$$

AREA OF TRIANGLE $= \frac{1}{2}|\vec{u}\times\vec{v}| = \frac{1}{2}\sqrt{25+100+1}$

$= \boxed{\frac{1}{2}\sqrt{126}}$ units^2

3. (7 pts) Find the angle of intersection of the two curves:

$$\mathbf{r}_1(t) = \langle t, 2-t, t^2 - 5t - 11\rangle \text{ and } \mathbf{r}_2(u) = \langle 5 - 2u, u - 4, u^3 + 4\rangle.$$

(Give your answer in degrees rounded to two digits after the decimal).

INTERSECT

① $t = 5 - 2u$
② $2 - t = u - 4$

① & ② $\Rightarrow$ $2-(5-2u) = u-4$

$-3+2u = u-4$

$u = -1$ $\Rightarrow$ $t = 7$

③ $t^2 - 5t - 11 = 49 - 35 - 11 = 3$ ✓

$u^3 + 4 = 3$ ✓

DIRECTIONS

$\vec{r}_1{}'(t) = \langle 1, -1, 2t-5\rangle$ $\qquad$ $\vec{r}_1{}'(7) = \langle 1, -1, 9\rangle$

$\vec{r}_2{}'(u) = \langle -2, 1, 3u^2\rangle$ $\qquad$ $\vec{r}_2{}'(-1) = \langle -2, 1, 3\rangle$

ANGLE

$$\langle 1,-1,9\rangle \cdot \langle -2,1,3\rangle = \sqrt{1+1+81}\sqrt{4+1+9}\cos\theta$$

$$\cos\theta = \frac{-2-1+27}{\sqrt{83}\sqrt{14}}$$

$$\theta = \cos^{-1}\left(\frac{24}{\sqrt{83}\sqrt{14}}\right) \approx 45.24654254^\circ$$

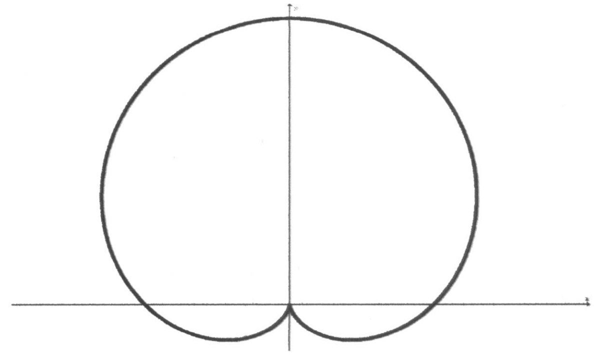
$\boxed{45.25^\circ}$

4. (7 pts) Find all (x, y) coordinates at which $r = \sin(\theta) + 1$ has a horizontal tangent.

$x = r\cos\theta$, $y = r\sin\theta$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

$$= \frac{\cos\theta\sin\theta + (\sin\theta+1)\cos\theta}{\cos\theta\cos\theta - (\sin\theta+1)\sin\theta} \stackrel{?}{=} 0$$



TOP $= 2\cos\theta\sin\theta + \cos\theta \stackrel{?}{=} 0$

$\cos\theta\,(2\sin\theta + 1) = 0$

$\cos\theta = 0$ OR $\sin(\theta) = -\frac{1}{2}$

$\theta = \frac{\pi}{2} + k\pi$ $\qquad \theta = \frac{7\pi}{6} + 2k\pi$, $\theta = \frac{11\pi}{6} + 2k\pi$

NOTE: AT $\theta = \frac{3\pi}{2} + 2k\pi$ $\frac{dy}{dx}$ UNDEFINED

$\theta = \frac{\pi}{2} \Rightarrow r(\frac{\pi}{2}) = 2 \Rightarrow (x,y) = (2\cdot 0, 2\cdot 1) = \boxed{(0,2)}$

$\theta = \frac{7\pi}{6} \Rightarrow r(\frac{7\pi}{6}) = \frac{1}{2} \Rightarrow (x,y) = (\frac{1}{2}\cdot\frac{-\sqrt{3}}{2}, \frac{1}{2}\cdot\frac{-1}{2}) = \boxed{(-\frac{\sqrt{3}}{4}, -\frac{1}{4})}$

$\theta = \frac{11\pi}{6} \Rightarrow r(\frac{11\pi}{6}) = \frac{1}{2} \Rightarrow (x,y) = (\frac{1}{2}\cdot\frac{\sqrt{3}}{2}, \frac{1}{2}(-\frac{1}{2})) = \boxed{(\frac{\sqrt{3}}{4}, -\frac{1}{4})}$

5. (5 pts) You are observing Dr. Loveless (in hopes of surprising him with a water balloon). He is going for a hike and his location is given by the position function

$$\mathbf{r}(t) = \left\langle t\cos(t), t\sin(t), 3\sqrt{t^2+1} \right\rangle$$

for $t \geq 0$, where t is in seconds and distances are in feet. Eliminate the parameter then circle the name that best characterizes the surface over which Dr. Loveless is hiking.

$x^2 + y^2 = t^2\cos^2(t) + t^2\sin^2(t) = t^2$

$z^2 = 9(t^2+1) \Rightarrow \frac{z^2}{9} = t^2 + 1 = x^2 + y^2 + 1$

$\Rightarrow -1 = x^2 + y^2 - \frac{z^2}{9}$

TRACES
- circles (DEPENDING ON z BIG ENOUGH)
- hyperbolas
- hyperbolas

Circle the name that is most appropriate for this surface:

CONE	SPHERE
ELLIPSOID	PARABOLIC CYLINDER
ELLIPTICAL CYLINDER	HYPERBOLIC CYLINDER
HYPERBOLOID OF ONE SHEET	HYPERBOLOID OF TWO SHEETS
ELLIPTIC PARABOLOID	HYPERBOLIC PARABOLOID
NONE OF THESE	

6. Consider the position function $\mathbf{r}(t) = \langle \ln(t), t^2 + 5, 3t\rangle$ for $t > 0$.

(a) (6 pts) Find where tangent line through the curve $\mathbf{r}(t)$ at $(0, 6, 3)$ intersects the xy-plane.

$(x, y, z) = (0, 6, 3) \Rightarrow t = 1$

$\vec{r}\,'(t) = \langle \frac{1}{t}, 2t, 3\rangle$

$\vec{r}\,'(1) = \langle 1, 2, 3\rangle$

TANGENT LINE : $x = 0 + u$, $y = 6 + 2u$, $z = 3 + 3u$

INTERSECT xy-plane $\Leftrightarrow$ $z = 0$ $\Leftrightarrow$ $3 + 3u = 0$

$u = -1$

$$\boxed{(x, y, z) = (-1, 4, 0)}$$

(b) (5 pts) Find all points on the curve $\mathbf{r}(t) = \langle \ln(t), t^2 + \cancel{1, t}\rangle$ at which the tangent line is orthogonal to the plane $\cancel{4}x + \cancel{2}y + 6z = 7$.

5 3t

1 8

WANT $\vec{r}\,'(t) = \langle \frac{1}{t}, 2t, 1\rangle$ TO BE PARALLEL

TO $\langle 4, 2, 6\rangle$.

A CONSTANT MULTIPLY

$\langle \frac{1}{t}, 2t, 1\rangle = k\langle 1, 8, 6\rangle$

① $\frac{1}{t} = k$

② $2t = 8k$

③ $3 = 6k \Longrightarrow k = \frac{1}{2}$ SO

① $\frac{1}{t} = \frac{1}{2} \Rightarrow t = 2$ ✓

② $2t = 4 \Rightarrow t = 2$ ✓

$t = 2$ $\vec{r}(2) = \langle \ln(2), 9, 6\rangle$

$$\boxed{(\ln(2), 9, 6)}$$