Exam 1 Review Overheads

Exam 1 details.

- 4 pages of questions
- Allowed a scientific calculator (**no graphing and no calculus abilities**)
- Allowed one **hand-written** 8.5 by 11 inch page of notes (double-sided)
- You must show your work on all problems.
- Covers 12.1-12.6, 10.1-10.3, 13.1-13.3. You should know all the facts and concepts covered in the homework for those sections.
- You have 50 minutes to complete the exam.

Exam 1 Basic Fact Sheet

1. Vector Operations: Sums, scalar multiples, dot products, cross products.
2. Vector Facts: checking orthogonality, checking parallel, angle between, area of parallelogram/triangle, projections.
3. Finding Line and Plane Equations.
4. Knowing basics of traces and knowing the 7 basic shapes and their names.
5. Working with Parametric Equations in $\mathbf{R}^2$:
   Slope, dy/dx, d^2y/dx^2, and parametric basics.
6. Working with Polar coordinates and polar curves:
   Plotting points, converting and dy/dx.
7. Working with Parametric Equations in $\mathbf{R}^3$:
   Tangent vector, unit tangent, tangent line, arc length, and curvature.

Basic Vector Facts:

1. $\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$

2. $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

3. $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}||\mathbf{v}| \cos(\theta)$

4. $\mathbf{u} \cdot \mathbf{v} = 0$ means orthogonal

5. $\mathbf{u} \times \mathbf{v} = |\mathbf{u}||\mathbf{v}| \sin(\theta)$

6. $\mathbf{u} \times \mathbf{v}$ is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$

7. $|\mathbf{u} \times \mathbf{v}| =$ parallelogram area

8. $\mathrm{comp}_{\mathbf{a}}(\mathbf{b}) = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|}$

9. $\mathbf{proj}_{\mathbf{a}}(\mathbf{b}) = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2} \mathbf{a}$

Basic Lines, Planes and Surfaces (assume all constants a, b and c are positive):

1. Lines: $x = x_0 + at$, $y = y_0 + bt$, $z = z_0 + ct$
   (x_0, y_0, z_0) = a point on the line
   $\langle a, b, c \rangle$ = a direction vector
2. Planes: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$
   (x_0, y_0, z_0) = a point on the plane
   $\langle a, b, c \rangle$ = a normal vector
3. Cylinder: One variable 'missing'
4. Elliptical Paraboloid: $z = ax^2 + by^2$ and
   Hyperbolic Paraboloid: $z = ax^2 - by^2$
5. Ellipsoid: $ax^2 + by^2 + cz^2 = 1$ and
   Cone: $z^2 = ax^2 + by^2$
6. Hyperboloid of One Sheet: $ax^2 + by^2 - cz^2 = 1$ and
   Hyperboloid of Two Sheets: $ax^2 + by^2 - cz^2 = -1$

Basic Parametric and Polar in $\mathbb{R}^2$:

1. $\langle \frac{dx}{dt}, \frac{dy}{dt} \rangle$ = a tangent vector

2. $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ and $\frac{d^2y}{dx^2} = \frac{d/dt(dy/dx)}{dx/dt}$

3. $x = r\cos(\theta)$ and $y = r\sin(\theta)$

4. $\tan(\theta) = \frac{y}{x}$ and $x^2 + y^2 = r^2$

5. $\frac{dy}{dx} = \frac{(dr/d\theta)\sin(\theta)+r\cos(\theta)}{(dr/d\theta)\cos(\theta)-r\sin(\theta)}$

Basic Parametric in $\mathbb{R}^3$:

1. $\mathbf{r}'(t) = \langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \rangle$

2. $\mathbf{r}''(t) = \langle \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2z}{dt^2} \rangle$

3. $\displaystyle\int \mathbf{r}(t)\,dt = \left\langle \int x(t)dt, \int y(t)dt, \int z(t)dt \right\rangle$

4. $\mathbf{T}(t) = \frac{1}{|\mathbf{r}'(t)|}\mathbf{r}(t)$

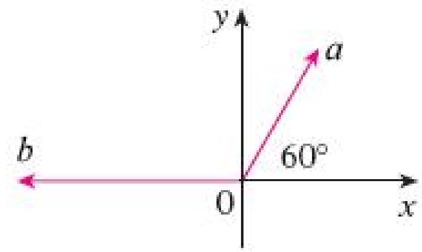
5. $s = \text{Arc Length} = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}\,dt$

6. $\kappa(t) = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$

7. For a function $y = f(x)$ in $\mathbb{R}^2$, the curvature formula simplifies to $\kappa(x) = \frac{|f''(x)|}{(1+(f'(x))^2)^{3/2}}$

Fall 2013 Exam:

1(a) The forces **a** and **b** are the pictured. If $|\mathbf{a}| = 80$ N and $|\mathbf{b}| = 100$ N, find the angle the **resultant** force makes with the positive x-axis.
(Give your answer rounded to the nearest degree).



(b) Find the center and radius of the sphere with points $P(x, y, z)$ such that the distance from P to $A(0, 0, 2)$ is triple the distance from P to $B(0, 0, 0)$.

2(a) Find the equation for the plane that contains the line $x = t$, $y = 1 - 2t$, $z = 4$ and the point $(3, -1, 5)$.

(b) Consider the line L_1 that goes through the points $(-3, 3, 0)$ and $(-1, 4, 6)$ and the line L_2 that is given by $x = 2 + t$, $y = 3 - 2t$, $z = 19 + 7t$. These lines are not parallel.

Are L_1 and L_2 intersecting or skew? Justify your answer by either finding the point of intersection or showing that there is no intersection point.

3(a) Find a vector $\mathbf{v}$ such that

1. $\mathbf{v}$ is parallel to the tangent line to $x = 6\ln(t-4)$, $y = t^2 - 3t$ at the point $(0, 10)$, and
2. $|\mathbf{v}| = 5$.

(b) The polar curve $r = 2 + \cos(3\theta)$ intersects the negative y-axis at only one point, P. Find the equation for the tangent line to the curve at this point P.

(Put your answer in the form $y = m(x - x_0) + y_0$).

4. Consider the vector function

$\mathbf{r}(t) = \langle t\cos(3t), t^2, t\sin(3t)\rangle$.

(a) Describe the surface of motion for the resulting parametric curve.
(Eliminate the parameter and give the specific name of the surface of motion).

(b) Find the parametric equations for the tangent line at $t = \pi$.

(c) Find the curvature at $t = 0$.

Spring 2013 Exam:

1(a) Consider the line through the points $P(1, 3, -2)$ and $Q(3, 5, 7)$. Find the (x, y, z) co-ordinates of the point at which this line intersects the xz-plane.

(b) Consider the **plane**, P, that contains the point $(1, -1, 2)$ and is the orthogonal to the line given by

$$L : \begin{cases} x = -3t \\ y = 2 + 7t \\ z = 5 - t \end{cases}$$

i. Find the equation for the plane, P.

ii. At what point (x, y, z) does this plane intersect the x-axis?

2(a) Assume **a** and **b** are nonzero three-dimensional vectors that are not parallel and are not perpendicular.

What is always true about the two given vectors:

i. $\mathbf{a} \times \mathbf{b}$ and $2\mathbf{b}$. orth./parallel/neither/depends

ii. $\mathbf{a} \times \mathbf{b}$ and $\mathbf{b} \times \mathbf{a}$. orth./parallel/neither/depends

iii. $\mathbf{proj_a}(\mathbf{b})$ and $\mathbf{b}$. orth./parallel/neither/depends

iv. $\mathbf{proj_a}(\mathbf{b})$ and $\frac{1}{|\mathbf{a}|}\mathbf{a}$. orth./parallel/neither/depends

v. $\mathbf{a} - \mathbf{b}$ and $\mathbf{b} - \mathbf{a}$. orth./parallel/neither/depends

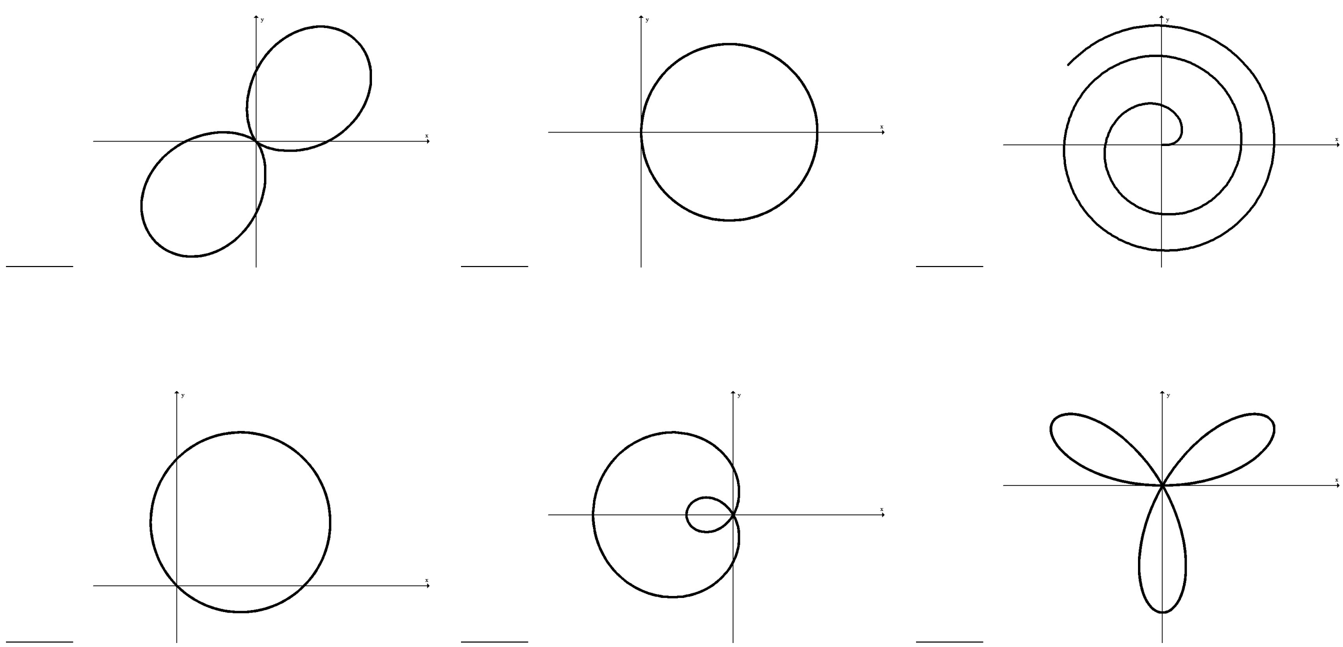
(b) Consider the three points $A(1, 3, 4)$, $B(0, 2, 1)$, $C(2, 3, 6)$.

i. Find the area of the triangle determined by the three points.

ii. For this same triangle, find the angle at the corner B.
(Give in degrees rounded to two places after the decimal).

3(a) In the blanks provided to the left of each graph, put the number of the polar equation that matches the graph in the xy-plane (two graphs will not be labeled).

1. $r = \sqrt{\theta}$ 2. $r = 1 - 2\cos(\theta)$ 3. $r = 1 + \sin(2\theta)$
4. $r = 9\cos(\theta)$



(b) Find the (x, y) coordinates of all points on the curve $r = 1 + \sin(2\theta)$ that intersect the line $y = x$.

4(a) Consider the vector function $\mathbf{r}(t) = \langle t^2 - 2t, t^3 - 4t\rangle$ and the corresponding parametric curve $x = t^2 - 2t$, $y = t^3 - 4t$.

i. Find the value of $\dfrac{d^2y}{dx^2}$ at $t = -1$.

ii. Find the value(s) of t at which the tangent line is parallel to the vector $\langle 1, 2\rangle$.

(b) Find parametric equations for the tangent line to the curve given by
$\mathbf{r}(t) = \langle 2\sin(3t), 3t, -2t\cos(t)\rangle$ at the time $t = \frac{\pi}{3}$.
(Give exact, simplified, numbers in your answer).