Local Max/Min for One Variable Function

A **critical value** is any number $x = c$ such that $f'(c) = 0$ or $f'(c)$ does not exist.

The Second Derivative Test:

If $x = c$ is a critical value, then

1. $f''(c) > 0 \Rightarrow x = c$ gives a local min.
2. $f''(c) < 0 \Rightarrow x = c$ gives a local max.
3. $f''(c) = 0 \Rightarrow$ inconclusive (other methods needed).

Local Max/Min for Two Variable Function

A **critical point** is any point $(x, y) = (a, b)$ such that $f_x(a, b) = 0$ AND $f_y(a, b) = 0$ (both) or $f_x(a, b)$ DNE or $f_y(a, b)$ DNE.

The Second Derivative Test:

If (a, b) is a critical point, then define

$$D = D(a, b) = f_{xx}(a, b)f_{yy}(a, b) - [f_{xy}(a, b)]^2.$$

1. $D > 0$, $f_{xx} > 0 \Rightarrow (a, b)$ gives a local min.
2. $D > 0$, $f_{xx} < 0 \Rightarrow (a, b)$ gives a local max.
3. $D < 0 \Rightarrow (a, b)$ gives a saddle point.
4. $D = 0 \Rightarrow$ inconclusive (use a contour map).

Global Max/Min for One Variable Function

To find abs. max/min of $f(x)$ on a closed interval:

1. Find critical numbers.
2. Evaluate $f(x)$ at the critical numbers.
3. Evaluate $f(x)$ at the endpoints.

   Biggest output = absolute max.

   Smallest output = absolute min.

Global Max/Min for Two Variable Function

To find abs. max/min of $f(x, y)$ on a closed region:

1. Find critical points.
2. Over each boundary curve:
   (a) Find xy equation.
   (b) Substitute boundary equation into $f(x, y)$ to get a one variable function for z.
   (c) Use Calculus 1 methods to find critical numbers and endpoints on that boundary.
3. Evaluate $f(x, y)$ at the critical points inside the region.
4. Evaluate $f(x, y)$ at the critical numbers and endpoints on each boundary.

   Biggest output = absolute max.

   Smallest output = absolute min.

Applied Optimization for single or multivariable fucntions

In applied problems, we have to set up the function to optimize. Here are things I always suggest:

1. VISUALIZE/LABEL: Draw a good picture and label **everything** with variables.
2. WHAT IS GIVEN?: Write down all the given **constraints**.
3. WHAT TO OPTIMIZE?: Write down a formula for that quantity. Then, using the given facts, find a function for the quantity that you want to optimize.
4. DOMAIN? Over what interval does the problem make sense
5. USE CALCULUS: Find the methods just discussed.
6. JUSTIFY/VERIFY: Make sure you actually did find the a max or min as desired.