MATH 126 EXAM 2 IS TUESDAY IN QUIZ SECTION.

YOU ARE ALLOWED:

- ONE 8.5 by 11 inch sheet of handwritten notes (front and back)
- A scientific calculator (no graphing, no cell phone)

COVERAGE: 13.3,13.4,

14.1, 14.3, 14.4, 14.7,

15.1-15.5

LENGTH: 4 pages in 50 minutes.

You will need to manage your time effectively, time may be a factor!

Chapter 13 - More with 3D curves:

If $\mathbf{r}(t) = < f(t), g(t), h(t) >$ = vector function, then

$\mathbf{r}'(t) = < f'(t), g'(t), h'(t) >$ = tangent vector

$\mathbf{T}(t) = \boldsymbol{r}'(t)/|\boldsymbol{r}'(t)|$ = unit tangent vector

$\mathbf{N}(t) = \boldsymbol{T}'(t)/|\boldsymbol{T}'(t)|$ = prin. unit normal vector

$\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$ = binormal vector

(Aside: r'(t) x r''(t) is also in the direction of the binormal, so you can compute B(t) by using r'(t) x r''(t) and making it a unit vector).

Normal plane, Osculating Plane

Curvature:

$\mathrm{K}(t) = d\boldsymbol{T}/ds = |\boldsymbol{T}'(t)|/|\boldsymbol{r}'(t)|$ = curvature

Another way to calculate curvature is

$\mathrm{K}(t) = |\boldsymbol{r}'(t) \times \boldsymbol{r}''(t)|/|\,\boldsymbol{r}'(t)|^3$

Velocity and Acceleration

If $\mathbf{r}(t) = < f(t), g(t), h(t) >$ = position function,

$\mathbf{v}(t) = \mathbf{r}'(t) = <f'(t), g'(t), h'(t)>$ = velocity

$|\mathbf{v}(t)|$ = speed function

$\mathbf{a}(t) = \mathbf{v}'(t) = \mathbf{r}''(t) = < f''(t), g''(t), h''(t) >$

= acceleration vector

If we are given the acceleration (or the forces because $\mathbf{F}(t) = m\mathbf{a}(t)$), we can integrate component-wise to get $\mathbf{v}(t)$ and $\mathbf{r}(t)$.

Be able to compute tangential and normal components of acceleration:

$$a_T = \boldsymbol{r}'(t) \cdot \boldsymbol{r}''(t) / |\boldsymbol{r}'(t)| = \mathrm{d}/\mathrm{dt}\, |\mathbf{v}(\mathrm{t})|$$

$$a_N = |\boldsymbol{r}'(t) \times \boldsymbol{r}''(t)| / |\boldsymbol{r}'(t)| = K(t)|\mathbf{v}(\mathrm{t})|^2$$

Chapter 14 Calculus on Surfaces:
The primary way to visualize a surface is to draw traces/level curves.
When we fix z=0, 1, 2, etc. and draw these together in the xy-plane we call this a **contour map**.

Be able to calculate partial derivatives and understand that
$f_x(x,y) = \partial z/\partial x$=slope in the positive x-direction
$f_y(x,y) = \partial z/\partial y$=slope in the positive y-direction
Be able to use the calculus 1 derivative rules in combination with the definition of partial derivatives.

The **tangent plane** to a surface at a point is given by

$$z - z_0 = f_x(x_0,y_0)(x - x_0) + f_y(x_0,y_0)(y - y_0)$$

If we label dz = z-z_0, dx = x-x_0, and dy = y-y_0, we call this the **total differential**, which would look like

$$dz = f_x(x_0,y_0)dx + f_y(x_0,y_0)dy, \text{ or}$$
$$dz = (\partial z/\partial x)dx + (\partial z/\partial y)dy$$

Yet a third way to view the tangent plane is as a linear approximation to z=f(x,y) for (x,y) near (x_0,y_0) that looks like:

$$z = L(x,y) = z_0 + f_x(x_0,y_0)(x - x_0) + f_y(x_0,y_0)(y - y_0)$$

so f(x,y) ≈ L(x,y) for (x,y) near (x_0,y_0).

Maximization/Minimization:
A **critical point** of z=f(x,y) is any point (x,y) such that $f_x(x,y) = 0$ AND $f_y(x,y) = 0$ simultaneously (or if one of the partial derivatives does not exist).

Second Derivative Test:
At each critical point, (a,b), calculate

$$D(a,b) = f_{xx}(a,b)\, f_{yy}(a,b) - [f_{xy}(a,b)]^2$$

(i) If D > 0 and $f_{xx}(a,b) > 0$, then f(a,b) is a **local minimum**.

(ii) If D > 0 and $f_{xx}(a,b) < 0$, then f(a,b) is a **local maximum**.

(iii) If D < 0, then (a,b) is a **saddle point**.

(iv) If D = 0, the test is inconclusive.

Absolute Maximum/Minimum Test:

1. Find critical points and evaluate $f(x,y)$ at the critical points in the domain.

2. i) For each boundary, write down the constraints (the equation for the boundary relating x and y).

ii) Substituting the constraints into the function $f(x,y)$ which will give you a function of one variable.

iii) Find the absolute max/min of the one variable function over the constraint using calculus 1 methods (find the critical values of the one variable and evaluate at the critical values and endpoints).

3. Biggest output = Absolute Max

   Smallest output = Absolute Min

Common Boundaries:

Rectangles: For each of the four sides, the constraint will have one of the variables (x or y) fixed and the other in an interval. Plug in the fixed value to z=f(x,y) and maximize/minimize the one variable function over the interval.

Triangles: For each side, find the equation of the line (y=mx+b), substitute this into z=f(x,y) and maximize/minimize a one input function over the interval.

Circles: The circle will look like $x^2 + y^2 = r^2$. So $y^2 = r^2 - x^2$ (and you also know that $-r \leq x \leq r$). Sometimes this is enough to substitute in, but you may need to break it down further into $y = +/- (r^2 - x^2)^{1/2}$. Plug this into to f(x,y) to get a one variable function.

Chapter 15 Double Integrals:
Be able to evaluate double integrals over

1. rectangles,
2. regions bounded between two curves,
3. polar regions.

Know how to draw the region and how to use this information to appropriately switch the order of integration which is sometimes necessary.

Be able to compute the necessary integrals for a center of mass problem.

Know the standard integration methods from Math 125.