Math 126 C - Spring 2010
Mid-Term Exam Number One
April 20, 2010
Answers

1. *Determine whether or not the line*

$$x = 4t - 7, y = 5t - 16, z = -2t + 14$$

and the line

$$x = t + 7, y = -3t - 7, z = 7t + 22$$

intersect. If they do, give the point of intersection.

The lines intersect at the point (5,-1,8), corresponding to $t = 3$ for the first line and $t = -2$ for the second line.

2. *Let P be the plane containing the points $(1, 5, 2)$, $(2, 3, 6)$ and $(7, 4, 1)$. Find the intersection of P with the y-axis.*

The plane P is given by

$$6x + 25y + 11z = 153.$$

The y-axis consists of all points satisfying

$$x = 0, z = 0$$

so the intersection with the y-axis is the point $x = 0, z = 0$ and

$$25y = 153$$

i.e., the point $(0, \frac{153}{25}, 0)$.

3. *Consider the polar curve*

$$r = \sin\theta \tan\theta.$$

(a) *Find an equivalent cartesian equation for this curve.*

(b) *The curve has a vertical asymptote. What is the equation of the asymptote?*

(a) An equivalent cartesian equation is

$$x(x^2 + y^2) = y^2.$$

(b) Rearranging, we have

$$y^2 = \frac{-x^3}{x - 1}$$

We see that the right-hand side is unbounded as x approaches 1; hence the curve has a vertical asymptote at $x = 1$.

4. *Let S be the surface in 3D consisting of all points which are twice as far from the z-axis as they are from the x-axis.*

   (a) *Give an example of a point on this surface, other than the origin.*

   (b) *Give an equation for this surface.*

   (c) *Describe this surface (if it is a quadric surface, categorizing it (i.e., ellipsoid, eliptic paraboloid, etc.) is sufficient).*

   (a) The point $(2, 0, 1)$ is such a point. (b) From

$$\sqrt{x^2 + y^2} = 2\sqrt{y^2 + z^2}$$

   we can arrive at

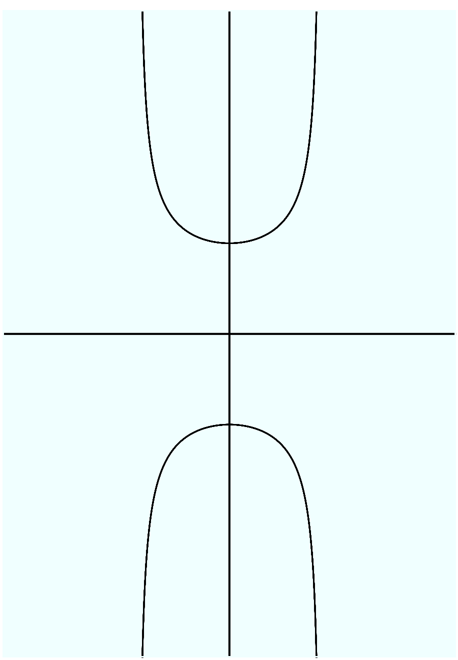
$$x^2 - 3y^2 - 4z^2 = 0$$

   (c) The surface is a quadric surface. Setting $y = 0$ or $z = 0$, we see the traces are pairs of degenerate hyperbolas. With x set to a constant, we see traces which are ellipses. We may conclude that the surface is a cone.

5. *Let P be the point in the first quadrant on the curve*

$$x = \cos t, y = \csc t$$

   such that the tangent line to the curve at P passes through the origin. Find the coordinates of P.



   By setting $\frac{dy}{dx} = \frac{y}{x}$ we find

$$\frac{\cos t}{\sin^3 t} = \frac{1}{\cos t \sin t}$$

which gives us

$$\cos^2 t = \sin^2 t.$$

This yields the solution

$$t = \frac{\pi}{4}$$

and so

$$P = \left(\frac{\sqrt{2}}{2}, \frac{2}{\sqrt{2}} \right).$$