15.3 and 15.4 Review

This review sheet discusses, in a very basic way, the key concepts from these sections. This review is not meant to be all inclusive, but hopefully it reminds you of some of the basics. Please notify me if you find any typos in this review.

1. **15.3 Double Integrals over General Regions**: Ultimately, this comes down to being able to describe different types of regions using inequailities involving x and y.

   (a) If we can describe the region R by writing $a \le x \le b$ and $f_1(x) \le y \le f_2(x)$, then we can evaluate the double integral via

   $$\int\int_R f(x,y)dA = \int_a^b \int_{f_1(x)}^{f_2(x)} f(x,y)dydx.$$

   (b) If we can describe the region R by writing $c \le y \le d$ and $g_1(y) \le x \le g_2(y)$, then we can evaluate the double integral via

   $$\int\int_R f(x,y)dA = \int_c^d \int_{g_1(y)}^{g_2(y)} f(x,y)dxdy.$$

   (c) WARNING: **You must always draw the region** AND **to switch the order of integration you must rediscribe the region when the variable are switch, you cannot do this automatically, you must look at the picture**! As a quick quiz, you should try to match up the integrals on the left with the integral describing the same region on the right. In other words, if you switch the dx and the dy what would the integral become (you must draw a picture to do each of these):

A. $\int_0^2 \int_0^x f(x,y)dydx$	a. $\int_0^1 \int_{y^2}^{\sqrt{y}} f(x,y)dxdy$
B. $\int_0^1 \int_0^{\sqrt{y}} f(x,y)dxdy$	b. $\int_0^2 \int_{-y+2}^{2} f(x,y)dydx$
C. $\int_0^1 \int_{x^2}^{\sqrt{x}} f(x,y)dydx$	c. $\int_0^2 \int_y^2 f(x,y)dxdy$
D. $\int_1^2 \int_0^{-x+2} f(x,y)dydx$	d. $\int_0^2 \int_0^{\frac{1}{2}x} f(x,y)dydx$
E. $\int_0^1 \int_{-\frac{1}{2}x+1}^{1} f(x,y)dxdy$	e. $\int_0^1 \int_{-\sqrt{1-y}}^{\sqrt{1-y}} f(x,y)dxdy$
F. $\int_0^1 \int_0^{-(x-1)^2+1} f(x,y)dydx$	f. $\int_0^3 \int_{\frac{1}{3}x}^{1} f(x,y)dydx$
G. $\int_0^1 \int_{2y}^{2} f(x,y)dxdy$	g. $\int_0^1 \int_1^{-y+2} f(x,y)dxdy$
H. $\int_0^1 \int_0^{3y} f(x,y)dxdy$	h. $\int_0^1 \int_{x^2}^{1} f(x,y)dydx$

   Make sure you know how to evaluate such integrals. The answers are on the next page.

2. **15.4 Double Integrals in Polar Coordinates**: Certain regions are much easier to integrate over if we change to polar coordinates.

   (a) Make sure that you can expresses a region in polar coordinates. To practice this make sure to look through all of the problems 1-16 in 15.4.

(b) To write an integral in polar coordinates. That is, we write the region R as $a \leq r \leq b$ and $\alpha \leq \theta \leq \beta$ and then we evaluate the integral:

$$\int\int_R f(x,y)dA = \int_\alpha^\beta \int_a^b f(r\cos(\theta), r\sin(\theta))r dr d\theta.$$

This says that you need to replace all x's with $r\cos(\theta)$, replace all y's with $r\sin(\theta)$ and replace dA with $rdrd\theta$. **Don't forget the r!**

(c) This method of double integration can be very useful, please make sure you know how to do it.

Unless I made a typo somewhere, the correct answers for the matching up on the previous page are as follows: A-c, B-h, C-a, D-g, E-b, F-e, G-d, H-f