Math 126 Exam 2 Review

[10.1-10.2] Parametric Curves

- Be able to sketch a parametric curve.
- Know how to eliminate the parameter.
- $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ = slope of tangent
- Arc Length, Area, Surface Area
- Be able to go back and forth between (x, y) & t.

[10.3] Polar Curves

- Be able to rewrite polar curves in Cartesian coordinates.
- $x = r\cos(\theta)$, $y = r\sin(\theta)$
  $r = x^2 + y^2$, $\tan(\theta) = \frac{y}{x}$
- $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\frac{dr}{d\theta}\sin(\theta) + r\cos(\theta)}{\frac{dr}{d\theta}\cos(\theta) - r\sin(\theta)}$
  $=$ slope of tangent

[12.5] Lines & Planes

- Lines: $\vec{r} = \vec{r}_0 + t\vec{v}$
- Planes: $\vec{n} \cdot (\vec{r} - \vec{r}_0) = 0$
  Be very comfortable with these.

[12.6] Intro to 3D graphing / shapes

- You will not have to draw 3D graphs.
- However, you should be able to graph 2D traces.

13.1 - 13.2 Vector Functions & Their Derivatives/Integrals

- Understand the basics of plotting vector functions.
- $\vec{r}'(t) = \langle f'(t), g'(t), h'(t) \rangle$
- $\int \vec{r}(t)\,dt = \langle \int f(t)\,dt, \int g(t)\,dt, \int h(t)\,dt \rangle$

13.3 Getting Info From 3D curve calculus

- Arc Length $= \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\,dt$
  or simply
  Arc Length $= \int_a^b |\vec{r}'(t)|\,dt$
- Be able to reparameterize in terms of arc length.
- Curvature (be able to compute it)

$$K = \left|\frac{d\vec{T}}{ds}\right| = \underbrace{\frac{|\vec{T}'(t)|}{|\vec{r}'(t)|}}_{\text{use this one unless you don't know } \vec{T}(t)} = \frac{|\vec{r}'(t) \times \vec{r}''(t)|}{|\vec{r}'(t)|^3}$$

- $\vec{T}(t) = \frac{\vec{r}'(t)}{|\vec{r}'(t)|}$ = unit tangent
  $\vec{N}(t) = \frac{\vec{T}'(t)}{|\vec{T}'(t)|}$ = principal unit tangent
  $\vec{B}(t) = \vec{T}(t) \times \vec{N}(t)$ = binormal

- Normal plane
  has $\vec{n} = \vec{r}'(t)$ as an orthogonal vector.

13.4 Acceleration, Velocity, and Speed.

- $\vec{r}(t)$ = position vector
  $\vec{v}(t) = \vec{r}'(t)$ = velocity vector
  $|\vec{v}(t)|$ = speed
  $\vec{a}(t) = \vec{r}''(t)$ = acceleration vector
- $\vec{F} = m\vec{a}$

- Be able to integrate acceleration and use initial conditions to find the position vector function.
- Tangential and Normal Components of acceleration

$$a_T = \text{tangential component} = \frac{d}{dt}(\text{speed}) = \frac{\vec{r}'(t) \cdot \vec{r}''(t)}{|\vec{r}'(t)|}$$

$$a_N = \text{normal component} = K(\text{speed})^2 = \frac{|\vec{r}'(t) \times \vec{r}''(t)|}{|\vec{r}'(t)|}$$

14.1 Multivariable Functions

- Be able to sketch the domain of a multivariable function
- Understand what level curves and contour maps are and how to draw them.

14.3 Partial Derivatives

- Be able to find first and second partial derivatives easily.
- Know what they give you.