Assignment 2 Homework Hints

Please notify me if you find any typos in this review.

1. **Taylor Notes HW 3 Problems 5, 6 and 7 -**

   - PROBLEM 5: In class, we found the pattern

     $$T_n(x) = \sum_{k=0}^{n} \frac{(-1)^k}{(2k+1)!} x^{2k+1} = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \cdots$$

     So you don't need to show this. Thus, all you have to do is the following: For any interval, $I = [-a, a]$, show that the error for $T_n(x)$ goes to zero as n goes to infinity. We did an almost identical problem in class for $f(x) = e^x$. Notice that you can use $|f^{(n+1)}(x)| \leq 1 = M$ as the same M for each n. So you just need to write what the error is for $T_n(x)$ and briefly state why it goes to zero.

   - PROBLEM 6: There are two parts:

     (a) Let $I = [-\frac{\pi}{2}, \frac{\pi}{2}]$. Note that $|f^{(n+1)}| \leq 1 = M$ for all n. Then try $n = 1, 2, 3, \ldots$ until the error in the error formula for this interval is less than 10^{-3}.

     (b) Let $I = [-\frac{\pi}{4}, \frac{\pi}{4}]$. Using the n found in part one, find the error on this new interval.

   - PROBLEM 7: I only want you to show that the error goes to zero as n goes to infinity on the interval $I = [-\frac{1}{2}, \frac{1}{2}]$. Note that $b = 0$. First try to find a pattern for $f^{(n+1)}(x)$. Then find M for $|f^{(n+1)}(x)| \leq M$ (Hint: it will always occur at the same endpoints.) This will give a formula for M that involves n. Now plug this formula for M into the error bound formula $|f(x) - T_n(x)| \leq \frac{M}{(n+1)!}|x - 0|^{n+1}$. Also plug in the endpoint for x. Finally simplify this expressing and see if you can verify that it goes to zero as $n \to \infty$.

2. **Taylor Notes HW 4** This is mostly up to you. You will need to play around with the 6 Taylor series discussed in class to get the desired function. In most cases you will be able to use straight substitution. However, you will need to use trig identities at least once (maybe twice) and you will need to use partial fractions at least once. Definitely make sure to read my week 2 review.

3. **Taylor Notes HW 5** This is very similar to HW 4, but, in addition, you will need to work with integrals/derivatives of Taylor series. Again you will likely need to use partial fractions at least once. You will have to do a couple of products as well. I suggest that you write out the series using "$\cdots$" and then seeing if you can identify a pattern. For instance:

   $$\begin{aligned} \tfrac{x^2}{1-x} &= x^2\left(\tfrac{1}{1-x}\right) \\ &= x^2\left(1 + x + x^2 + x^3 + \cdots\right) \\ &= x^2 + x^3 + x^4 + x^5 + \ldots \\ &= \textstyle\sum_{k=2}^{\infty} x^k \end{aligned}$$

   Keep experimenting!