Math 126 Homework 13.3/49 and 13.4/38 Hints

Problem 49 on 13.3 and problem 38 on 13.4 of the homework are trying to get you to practice a bit with vector and derivative operations relating to the tangent, principal unit normal, and binormal vectors. However, it is quite challenging without a few helpful hints. Here are hints to get you started:

Problem 13.3/49

1. Part (a): Start with $|\mathbf{B}| = 1$ and derive that $\dfrac{d\mathbf{B}}{ds} \cdot \mathbf{B} = 0$. Use the exact same technique that we used in class to derive that $\mathbf{T}'(t) \cdot \mathbf{T}(t) = 0$. As an extra challenge, I suggest you try to give an argument for why $|\mathbf{B}| = 1$.

2. Part (b): Start with $\mathbf{B} = \mathbf{T} \times \mathbf{N}$ and show that $\dfrac{d\mathbf{B}}{ds} = \dfrac{\mathbf{T} \times \mathbf{N}'}{|\mathbf{r}'(t)|}$. (You will need to use the cross product rule). From this, explain why $\frac{d\mathbf{B}}{ds}$ is orthogonal to $\mathbf{T}$.

   You will need to use the following facts: (i) $\dfrac{d\mathbf{B}}{ds} = \dfrac{d\mathbf{B}/dt}{ds/dt}$, (ii) $\mathbf{T}' \times \mathbf{N} = \mathbf{0}$ because they are parallel, and (iii) the cross product of two vectors is orthogonal to each vector.

3. Part (c): From parts (a) and (b), you know that $\dfrac{d\mathbf{B}}{ds}$ is orthogonal to both $\mathbf{B}$ and $\mathbf{T}$. From this you can deduce a relationship between $\dfrac{d\mathbf{B}}{ds}$ and $\mathbf{N}$.

4. Part (d): A plane curve is a curve that is always traveling on the same plane. Thus, $\mathbf{B}$ is always the same vector. That is, $\mathbf{B}$ is a constant unit vector with respect to s. Hence, $\dfrac{d\mathbf{B}}{ds} = 0$. Now use part (c) to deduce the result.

Problem 13.4/38

Start with the defined relationship $\mathbf{L}(t) = m\mathbf{r}(t) \times \mathbf{v}(t)$ and compute $\mathbf{L}'(t)$ by using the cross product rule. (Note that $\mathbf{v}(t) \times \mathbf{v}(t) = \mathbf{0}$ because they are parallel vectors).