Ch. 12 Vector Ops / Lines / Planes

- $\langle a, b\rangle \cdot \langle c, d\rangle = ac + bd$
- $\langle a, b, c\rangle \cdot \langle d, e, f\rangle = ad + be + cf$
- $\langle a, b, c\rangle \times \langle d, e, f\rangle$

  $= \langle bf - ce, cd - af, ae - bd\rangle$

  Remember the computational method discussed in class.

- $\boxed{\text{Lines}}$

  If $\vec{v} = \langle a, b, c\rangle$ = a vector parallel to the line (direction vector)

  (x_0, y_0, z_0) = a point on the line

  then the equation for points (x, y, z) on the line are given by

  $\boxed{\text{vector form}}$

  $\boxed{\text{I}}$ $\langle x, y, z\rangle = \langle x_0, y_0, z_0\rangle + t\langle a, b, c\rangle$

  $\boxed{\text{parametric form}}$

  $\boxed{\text{II}}$

  $$x = x_0 + at$$
  $$y = y_0 + bt$$
  $$z = z_0 + ct$$

  $\boxed{\text{symmetric form}}$

  $\boxed{\text{III}}$ $$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

$\boxed{\text{Aside}}$ Tangent Line at (x_0, y_0, z_0) for $\vec{r}(t) = \langle f(t), g(t), h(t)\rangle$

Take $\vec{v} = \vec{r}\,'(t)$

$\vec{v} = \langle f'(t), g'(t), h'(t)\rangle$

Planes

If $\vec{n} = \langle a, b, c \rangle$ = a normal vector to the plane
= orthogonal to any vector parallel to the plane.

(x_0, y_0, z_0) = any point on the plane

then the equation for points (x, y, z) on the plane are given by

vector form

I $\langle a, b, c \rangle \cdot (\langle x, y, z \rangle - \langle x_0, y_0, z_0 \rangle) = 0$

scalar form

II

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

Aside

Normal Plane at a point (x_0, y_0, z_0) for $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$

- Take $\vec{n} = \vec{r}\,'(t)$

$\vec{n} = \langle f'(t), g'(t), h'(t) \rangle$

Tangent Plane at a point (x_0, y_0, z_0) for a surface $z = f(x, y)$

- Take $\vec{n} = \langle 1, -f_x(x_0, y_0), -f_y(x_0, y_0) \rangle$

Ch. 10 Parametric AND Polar

- Know how to work with parametric curves:

$$x = f(t) \qquad y = g(t)$$

which can be written as $\langle f(t), g(t)\rangle = \vec{r}(t)$

- As with curves in 3D,

$$\vec{r}\,'(t) = \langle f'(t), g'(t)\rangle = \text{velocity vector}$$
$$\vec{r}\,''(t) = \langle f''(t), g''(t)\rangle = \text{acceleration vector}$$

- slope of the tangent line $= \dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt}$

- Understand Polar Curves

$$x = r\cos(\theta) \qquad r^2 = x^2 + y^2$$
$$y = r\sin(\theta) \qquad \tan(\theta) = y/x$$

Be able to go back and forth between polar and cartesian coordinates.

- slope of the tangent line $= \dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta} = \dfrac{\frac{dr}{d\theta}\sin(\theta) + r\cos(\theta)}{\frac{dr}{d\theta}\cos(\theta) - r\sin(\theta)}$

Recall: The equation of a tangent line is of the form

$$y = \underset{\uparrow}{m}x + b$$

$\leftarrow$ slope of tangent line $= \dfrac{dy}{dx}$