Exam 1 Basic Fact Sheet

Basic Vector Facts

$\lvert\mathbf{v}\rvert = \sqrt{v_1^2 + v_2^2 + v_3^2}$	$c\mathbf{v} = \langle cv_1, cv_2, cv_3 \rangle$	$\frac{1}{\lvert\mathbf{v}\rvert}\mathbf{v} =$ ‘unit vector in direction of $\mathbf{v}$
$\mathbf{u} + \mathbf{v} = \langle u_1 + v_1, u_2 + v_2, u_3 + v_3 \rangle$	$\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2 + u_3v_3$	$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$
$\mathbf{u} \cdot \mathbf{v} = \lvert\mathbf{u}\rvert\lvert\mathbf{v}\rvert \cos(\theta)$	$\mathbf{u} \cdot \mathbf{v} = 0$ means orthogonal	θ is the angle if drawn tail to tail
$\lvert\mathbf{u} \times \mathbf{v}\rvert = \lvert\mathbf{u}\rvert\lvert\mathbf{v}\rvert \sin(\theta)$	$\mathbf{u} \times \mathbf{v}$ is orthogonal to both	$\lvert\mathbf{u} \times \mathbf{v}\rvert =$ parallelogram area
$\text{comp}_{\mathbf{a}}(\mathbf{b}) = \frac{\mathbf{a}\cdot\mathbf{b}}{\lvert\mathbf{a}\rvert}$	$\mathbf{proj}_{\mathbf{a}}(\mathbf{b}) = \frac{\mathbf{a}\cdot\mathbf{b}}{\lvert\mathbf{a}\rvert^2}\mathbf{a}$	

Basic Lines, Planes and Surfaces (assume all constants a, b and c are positive)

Lines: $x = x_0 + at$, $y = y_0 + bt$, $z = z_0 + ct$	$(x_0, y_0, z_0) =$ a point on the line $\langle a, b, c \rangle =$ a direction vector
Planes: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$	$(x_0, y_0, z_0) =$ a point on the plane $\langle a, b, c \rangle =$ a normal vector
Cylinder: One variable ‘missing’	Know basics of traces
Elliptical/Circular Paraboloid: $z = ax^2 + by^2$	Hyperbolic Paraboloid: $z = ax^2 - by^2$
Ellipsoid/Sphere: $ax^2 + by^2 + cz^2 = 1$	Elliptical/Circular Cone: $z^2 = ax^2 + by^2$
Hyperboloid of One Sheet: $ax^2 + by^2 - cz^2 = 1$	Hyperboloid of Two Sheets: $ax^2 + by^2 - cz^2 = -1$

Basic Curves in $\mathbb{R}^3$

$\mathbf{r}'(t) = \langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \rangle$	$\mathbf{r}''(t) = \langle \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2z}{dt^2} \rangle$
$\displaystyle\int \mathbf{r}(t)\,dt = \left\langle \int x(t)dt, \int y(t)dt, \int z(t)dt \right\rangle$	Note: There are three constants of integration.
Arc Length $= \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}\,dt$	$s(t) = \int_0^t \sqrt{(x'(u))^2 + (y'(u))^2 + (z'(u))^2}\,du$
$\kappa(t) = \frac{\lvert\mathbf{r}'(t) \times \mathbf{r}''(t)\rvert}{\lvert\mathbf{r}'(t)\rvert^3}$	$\kappa(x) = \frac{\lvert f''(x)\rvert}{(1+f'(x)^2)^{3/2}} =$ ‘2D curvature’
$\mathbf{r}'(t) = \mathbf{v}(t) =$ velocity vector	$\lvert\mathbf{r}'(t)\rvert = \lvert\mathbf{v}(t)\rvert =$ speed
$\mathbf{r}''(t) = \mathbf{a}(t) =$ acceleration	$\mathbf{r}(t) = \int \mathbf{v}(t)\,dt$ and $\mathbf{v}(t) = \int \mathbf{a}(t)\,dt$
$\mathbf{T}(t) = \frac{1}{\lvert\mathbf{r}'(t)\rvert}\mathbf{r}'(t) =$ unit tangent	$\mathbf{N}(t) = \frac{1}{\lvert\mathbf{T}'(t)\rvert}\mathbf{T}'(t) =$ principal unit normal
$\mathbf{B}(t) = \mathbf{T} \times \mathbf{N} =$ binormal	$\mathbf{r}'(t) \times \mathbf{r}''(t) =$ a vector parallel to $\mathbf{B}$
$a_T = \frac{\mathbf{r}'(t)\cdot\mathbf{r}''(t)}{\lvert\mathbf{r}'(t)\rvert}$	$a_N = \frac{\lvert\mathbf{r}'(t)\times\mathbf{r}''(t)\rvert}{\lvert\mathbf{r}'(t)\rvert}$