SPRING 2020 — EXAM I (CH. 12)

QUESTION 1 (a) $\underbrace{\vec{u} \times \vec{v}}_{\text{vector}} + \underbrace{\vec{v}}_{\text{vector}} = \boxed{\text{VECTOR}}$

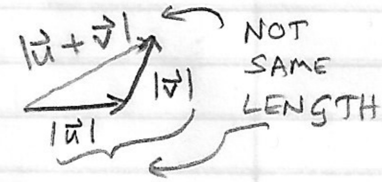
(b)

$\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$ $\boxed{\text{TRUE}}$

$|\vec{u} + \vec{v}| = |\vec{u}| + |\vec{v}|$ $\boxed{\text{FALSE}}$

$\vec{u} \times \vec{v} = \vec{v} \times \vec{u}$ $\boxed{\text{FALSE}}$

$\vec{v} \cdot \vec{v} = |\vec{v}|^2$ $\boxed{\text{TRUE}}$



(c) TWO LINES EITHER INTERSECT OR ARE PARALLEL $\boxed{\text{FALSE}}$

could be skew

TWO PLANES BOTH PERPENDICULAR TO A THIRD PLANE ARE PARALLEL $\boxed{\text{FALSE}}$

look at two intersecting walls and the ceiling in the room you are in

TWO PLANES EITHER INTERSECT OR ARE PARALLEL $\boxed{\text{TRUE}}$

TWO LINES PERPENDICULAR TO THE SAME PLANE ARE PARALLEL $\boxed{\text{TRUE}}$

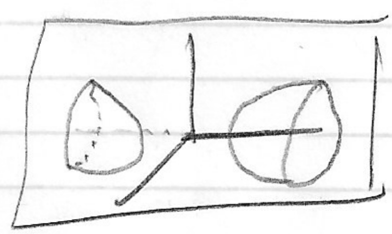
(d) IF POINT IS $(6, 4, 8)$, THEN ANSWER IS

CLOSEST TO LEAVING FIRST OCTANT

$$\boxed{(x-6)^2 + (y-4)^2 + (z-8)^2 = 16} \leftarrow 4^2$$

(e) $-x^2 + y^2 - z^2 = 1 \Rightarrow x^2 - y^2 + z^2 = -1$

$\boxed{\text{HYPERBOLOID OF TWO SHEETS}}$



2 POINTS EACH, NO PARTIAL CREDIT

QUESTION 2 (a) $A(1,0,-1)$, $B(5,-4,0)$, $C(1,5,4)$

3 PTS

$$\vec{CA} = \langle 1-1, 0-5, -1-4\rangle = \langle 0,-5,-5\rangle = \vec{u}$$
$$\vec{CB} = \langle 5-1, -4-5, 0-4\rangle = \langle 4,-9,-4\rangle = \vec{v}$$
$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}|\,|\vec{v}|} = \frac{0+45+20}{\sqrt{25+25}\,\sqrt{16+81+16}} = \frac{65}{\sqrt{50}\,\sqrt{113}}$$
$$\theta = \cos^{-1}\left(\frac{65}{\sqrt{50}\,\sqrt{113}}\right) \approx 30.15^\circ \approx \boxed{30^\circ}$$

(b) $y = \frac{1}{3}x^3 \Rightarrow y' = x^2 \Rightarrow$ "slope at $x=2$ is" $= y'(2) = 4$

3 PTS

$\underbrace{\langle 1,4\rangle}_{\text{PARALLEL TO TANGENT}}$

MAKES IT LENGTH 6

$$\boxed{\frac{6}{\sqrt{17}}\langle 1,4\rangle \quad \text{OR} \quad \frac{-6}{\sqrt{17}}\langle 1,4\rangle}$$

MAKES IT LENGTH 1

(c) $\underbrace{\text{"DIST. FROM } (x,y,z) \text{ TO } x\text{-AXIS"}}$ $= 4 \cdot$ $\underbrace{\text{"DIST. FROM } (x,y,z) \text{ TO } yz\text{-PLANE"}}$

3 PTS

"DIST. FROM (x,y,z) TO $(x,0,0)$" $= 4$ "DIST. FROM (x,y,z) to $(0,y,z)$"

$$\sqrt{(x-x)^2+y^2+z^2} = 4\sqrt{(x-0)^2+(y-y)^2+(z-z)^2}$$
$$\sqrt{y^2+z^2} = 4\sqrt{x^2}$$
$$\boxed{y^2+z^2 = 16x^2}$$

1 PT $\rightarrow$ $\boxed{\text{CONE}}$

QUESTION $\boxed{3}$ (a) LINE THRU $P(1,0,1)$ AND $Q(3,-4,4)$ INTERSECT $x+y+z=4$?

$$\left.\begin{aligned} x &= 1 + 2t \\ y &= 0 - 4t \\ z &= 1 + 3t \end{aligned}\right\}$$

$\uparrow$ P $\quad \uparrow$ $\overrightarrow{PQ}$

4 PTS

$$x + y + z = 4$$
$$(1+2t) + (-4t) + (1+3t) = 4$$
$$2 + t = 4$$
$$t = 2$$

$$\boxed{(x,y,z) = (5,-8,7)}$$

(b) CONTAINS $P(-3,1,3)$ AND LINE OF INTERSECTION OF $x+y-z=4$ AND $4x-y+5z=1$.

FIND TWO POINTS ON INTERSECTION!

① $x+y-z=4 \rightarrow y = 4-x+z$

② $4x-y+5z=1$

6 PTS

COMBINE: $5x+4z=5$

$z=0 \Rightarrow x=1 \Rightarrow y=3 \quad Q(1,3,0)$

$x=0 \Rightarrow z=\frac{5}{4} \Rightarrow y=4+\frac{5}{4}=\frac{21}{4} \quad R\left(0,\frac{21}{4},\frac{5}{4}\right)$

$\frac{5}{4}-3=\frac{5}{4}-\frac{12}{4}=-\frac{7}{4}$

$\overrightarrow{PQ} = \langle 4, 2, -3\rangle$

$\overrightarrow{PR} = \langle 3, \frac{17}{4}, -\frac{7}{4}\rangle$ ← CAN SCALE BY FACTOR OF 4 TO SIMPLIFY IF YOU WANT

$4\overrightarrow{PR} = \langle 12, 17, -7\rangle$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 2 & -3 \\ 12 & 17 & -7 \end{vmatrix} = (-14 - -51)\vec{i} - (-28 - -36)\vec{j} + (68-24)\vec{k}$$
$$= \langle 37, -8, 44\rangle$$

$$\boxed{37(x+3) - 8(y-1) + 44(z-3) = 0}$$

$$37x - 8y + 44z = 13$$

if expanded and simplified.