Worksheet 3: Surfaces and Parametric Facts

This worksheet reviews and summarizes several topics that you are learning right now (or are about to learn). Use this time to discuss these new topics in groups and to clear up confusion with your TA.

1. *Surfaces:* Consider the surface given by $x + y^2 - z^2 = 4$.

   - If $x = k$ is fixed, identify the trace.
   - If $y = k$ is fixed, identify the trace.
   - If $z = k$ is fixed, identify the trace.
   - What is the name of this shape?

2. *Basic Parametric:* Consider the curve given by $x = t$, $y = t\sin(\pi t)$, $z = t\cos(\pi t)$. Eliminate the parameter in order to get a surface over which this motion is occurring (you should know the name of this surface).

3. *Basic Parametric:* Two objects are traveling through space with vector functions given by $\mathbf{r}_1(t) = \langle t, 5t, t^2 \rangle$ and $\mathbf{r}_2(t) = \langle 5 - t, 7t + 1, t^3 + 1 \rangle$.

   - Find all points at which their **paths** intersect. (Hint: Be careful with parameters).
   - Do the objects ever **collide**? (If so, find the time when they collide. If not, explain why.)

4. *Parametric Calculus:* For the curve given by $\mathbf{r}(t) = \langle t^2 + 1, t^3, 1 - 5t \rangle$.

   - Find $r'(t)$.
   - Find $r'(1)$ and $T(1)$.
   - Find parametric equations for the tangent line to the curve at $t = 1$.
   - *Challenge question*: Find the acute angle at which the curve intersects the xy-plane (i.e. the angle it makes with the ground). Give your answer in degrees.