1. (12 pts) **The [redacted] parts below are not related.**

6 (a) Give the equation for the tangent plane to $f(x,y) = \dfrac{e^{5y}x^3}{1+8y}$ at $(x,y) = (2,0)$

$$f_x = \frac{3e^{5y}x^2}{1+8y} \Rightarrow f_x(2,0) = \frac{3e^{0}(2)^2}{1+8(0)} = 12$$

$$f_y = \frac{(1+8y)5e^{5y}x^3 - 8e^{5y}x^3}{(1+8y)^2} \Rightarrow f_y(2,0) = \frac{(1)(5)e^{0}(2)^3 - 8e^{0}(2)^3}{(1)^2}$$

$$= \frac{40-64}{1} = -24$$

$$f(2,0) = \frac{e^{0}(2)^3}{1+8(0)} = 8$$

$$\boxed{z - 8 = 12(x-2) - 24y}$$

6 (b) Find $\dfrac{\partial z}{\partial x}$ at the point $(x,y,z) = (1,0,2)$ for the surface implicitly defined by

$$4xz - z^3 = \sin(\pi x + 3y) + 5\ln(1+y).$$

$$4z + 4x\frac{\partial z}{\partial x} - 3z^2\frac{\partial z}{\partial x} = \pi\cos(\pi x + 3y)$$

$$(1,0,2) \Rightarrow 4(2) + 4(1)\frac{\partial z}{\partial x} - 3(2)^2\frac{\partial z}{\partial x} = \pi\cos(\pi)$$

$$\Rightarrow 4\frac{\partial z}{\partial x} - 12\frac{\partial z}{\partial x} = -\pi - 8$$

$$-8\frac{\partial z}{\partial x} = -\pi - 8$$

$$\boxed{\frac{\partial z}{\partial x} = \frac{\pi + 8}{8} = \frac{\pi}{8} + 1}$$

ASIDE:

$$\frac{\partial z}{\partial x} = \frac{\pi\cos(\pi x + 3y) - 4z}{4x - 3z^2}$$

10

2. (10 pts) Find and classify all critical point(s) for $z = f(x, y) = x^3 - x^2y + y^2 - 2y$.
Clearly label whether each critical point, $(x, y)$, gives a local max, local min or saddle point.
Show all appropriate steps of the second derivative test.
(You do NOT need to find the corresponding $z$-values, just give the points, $(x, y)$).

(i) $f_x = 3x^2 - 2xy \overset{?}{=} 0 \Rightarrow x(3x - 2y) = 0$

$\Rightarrow x = 0$ or $3x - 2y = 0$

(ii) $f_y = -x^2 + 2y - 2 \overset{?}{=} 0$

$x = 0 \Rightarrow -(0)^2 + 2y - 2 = 0 \Rightarrow y = 1 \Rightarrow \boxed{(0,1)}$

$3x - 2y = 0 \Rightarrow y = \frac{3}{2}x \Rightarrow -x^2 + 3x - 2 = 0 \Rightarrow -(x-2)(x-1) = 0$

$x = 1$ or $x = 2$

$\downarrow \quad \downarrow$

$y = \frac{3}{2} \quad y = 3$

$\boxed{(1, 3/2)} \quad \boxed{(2, 3)}$

$f_{xx} = 6x - 2y \quad f_{yy} = 2 \quad f_{xy} = -2x$

$\boxed{(0,1)} \Rightarrow f_{xx} = -2,\ f_{yy} = 2,\ f_{xy} = 0,\ D = -4 < 0$ } $\boxed{\text{SADDLE POINT}}$

$\boxed{(1, \frac{3}{2})} \Rightarrow f_{xx} = 3,\ f_{yy} = 2,\ f_{xy} = -2,\ D = 6 - (-2)^2 = 2 > 0$ } $\boxed{\text{LOCAL MIN}}$

$\boxed{(2,3)} \Rightarrow f_{xx} = 6,\ f_{yy} = 2,\ f_{xy} = -4,\ D = 12 - (-4)^2 = -4 < 0$ } $\boxed{\text{SADDLE POINT}}$

7

3. (14 pts) **The two parts below are not related.**

(a) Find the absolute maximum and minimum of $f(x,y) = 3y - xy$ over the region bounded by $y = x^2$, $y = 0$, and $x = 4$.

$\boxed{\text{BOUNDARIES}}$

$\boxed{A}$ $y = 0 \Rightarrow z = f(x,0) = 0 \leftarrow$ CONSTANT

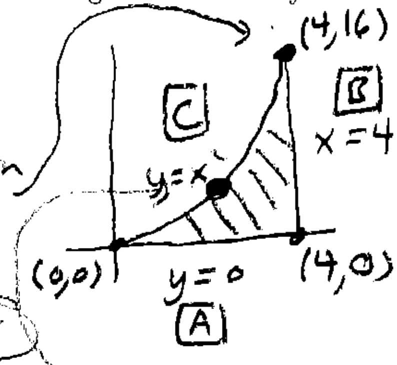
$\boxed{B}$ $x = 4 \Rightarrow z = f(4,y) = 3y - 4y = -y \leftarrow$ LINEAR, ONLY NEED TO CONSIDER ENDPOINTS

$\boxed{C}$ $y = x^2 \Rightarrow z = f(x,x^2) = 3x^2 - x^3$

$z' = 6x - 3x^2 \stackrel{?}{=} 0 \Rightarrow 3x(2-x) = 0$

$x = 0$ or $x = 2$

$y = 4$ $(2,4)$




$\boxed{\text{INSIDE}}$

$f_x = -y = 0$ $\quad (3,0) \leftarrow$ ON BOUNDARY

$f_y = 3 - x = 0$

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$(0,0) \Rightarrow z = 0$

$(4,0) \Rightarrow z = 0$

$(4,16) \Rightarrow z = 3(16) - 4(16) = \boxed{-16 \leftarrow \text{ABS MIN}}$

$(2,4) \Rightarrow z = 3(4) - 2(4) = \boxed{4 \leftarrow \text{ABS MAX}}$

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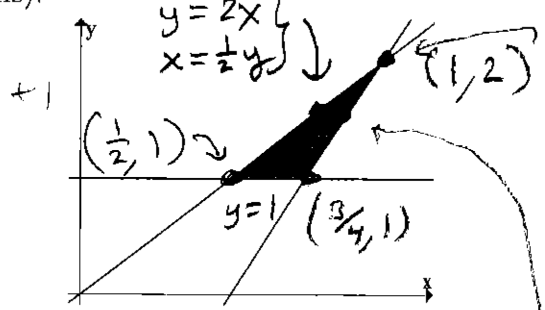
(b) Let $D$ be the region in the $xy$-plane bounded by $y = 2x$, $y = 4x - 2$ and $y = 1$ (shown below). Set up the integral $\iint_D g(x,y)dA$ in BOTH ways $dxdy$ and $dydx$. Do NOT evaluate. (Note: One way will require you to split up into two regions).

$2x = 4x - 2 \Rightarrow 2x = 2 \Rightarrow x = 1$

$1 = 2x \Rightarrow x = 1/2$

$1 = 4x - 2 \Rightarrow x = 3/4$

+1




$y = 4x - 2$

$y + 2 = 4x$

$x = \frac{1}{4}(y+2)$

+7 $\boxed{\int_1^2 \int_{\frac{1}{2}y}^{\frac{1}{4}(y+2)} g(x,y)\,dx\,dy}$

AND

+7 $\boxed{\int_{\frac{1}{2}}^{\frac{3}{4}} \int_1^{2x} g(x,y)\,dy\,dx + \int_{\frac{3}{4}}^{1} \int_{4x-2}^{2x} g(x,y)\,dy\,dx}$

4. (14 pts) **The two problems below are not related.**

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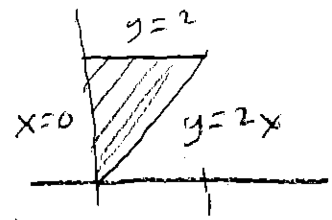
(a) Find the volume of the solid below the plane $z = 10$, above the paraboloid $z = 6 - 3x^2 - 3y^2$, and enclosed by the planes $x = 0$, $y = 2$ and $y = 2x$.

$$\iint_D 10\,dA - \iint_D 6 - 3x^2 - 3y^2\,dA$$

$$\iint_D 4 + 3x^2 + 3y^2\,dA$$




$$\int_0^1 \int_{2x}^{2} 4 + 3x^2 + 3y^2\,dy\,dx \qquad \text{Or} \qquad \int_0^2 \int_0^{\frac{1}{2}y} 4 + 3x^2 + 3y^2\,dx\,dy$$

$$\int_0^1 4y + 3x^2y + y^3 \Big|_{2x}^{2} dx$$

$$= \int_0^1 (8 + 6x^2 + 8) - (8x + 6x^3 + 8x^3)\,dx$$

$$= \int_0^1 16 + 6x^2 - 8x - 14x^3\,dx$$

$$= 16x + 2x^3 - 4x^2 - \frac{14}{4}x^4 \Big|_0^1$$

$$= 16 + 2 - 4 - \frac{7}{2} = 14 - \frac{7}{2} = \boxed{\frac{21}{2} = 10.5}$$

7

(b) Rewrite the following double integral in polar coordinates, then evaluate: $\displaystyle\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} x^2\,dx\,dy$.

$$\int_0^{\pi/4} \int_0^2 r^2\cos^2\theta \; r\,dr\,d\theta$$

$$\int_0^{\pi/4} \cos^2\theta \; \frac{1}{4}r^4 \Big|_0^2 \, d\theta$$

$$4 \int_0^{\pi/4} \frac{1}{2}(1 + \cos(2\theta))\,d\theta$$

$$2\left[\theta + \frac{1}{2}\sin(2\theta)\Big|_0^{\pi/4}\right]$$

$$\boxed{2\left(\pi/4 + \frac{1}{2}\right) = \frac{\pi}{2} + 1} = \frac{\pi + 2}{2}$$