

### Exam 3 - Ch. 14 Review Overheads

#### Rules:

- A mult. choice prob. - several parts (no partial). Two problems where you show work 2-3 parts each.
- A basic scientific calculator allowed (no graphing or calculus calculator). Allowed one **hand-written** 8.5 by 11 inch page of notes (double-sided)
- Covers 14.1, 14.3, 14.4, 14.7. Know all facts and concepts covered in lecture and homework.
- You have 60 min. to complete exam. Exam opens at 12:00pm and closes at 3pm (start before 2pm to get full time). You must SUBMIT each of your final answers within the exam.
- Upload handwritten work on the exam itself, we prefer you do this, but if you run out of time or have technical problems uploading, then uploading them in the dropbox assignment immediately after exam is okay. Your handwritten work must be a pdf, jpg, png or gif file (NO HEIC files).

#### Review:

#### Ch. 14 - Slopes on Surfaces - Quick Summary

$z = f(a, b)$ = height above $xy$ -plane at $(a, b)$	Be able to find and graph the domain. Know the basics on level curves/contour maps
$f_x(a, b) = \frac{\partial z}{\partial x}$ = slope in $x$ -direction at $(a, b)$ $f_y(a, b) = \frac{\partial z}{\partial y}$ = slope in $y$ -direction at $(a, b)$	$\langle 1, 0, f_x(a, b) \rangle$ is a tangent vector in $x$ -direction $\langle 0, 1, f_y(a, b) \rangle$ is a tangent vector in $y$ -direction
$z = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b)$	Tangent plane or Linear Approximation at $(a, b)$
$f_{xx}(a, b) = \frac{\partial^2 z}{\partial x^2}$ = concavity in $x$ -direction at $(a, b)$ $f_{yy}(a, b) = \frac{\partial^2 z}{\partial y^2}$ = concavity in $y$ -direction at $(a, b)$	$f_{xy}(a, b) = \frac{\partial^2 z}{\partial y \partial x}$ = mixed second partial at $(a, b)$ $f_{xy} = f_{yx}$ (Clairaut's Theorem)
$D = f_{xx} \cdot f_{yy} - (f_{xy})^2$ = measure of concavity	at a critical point $(a, b)$ ... $D < 0$ means concavity changes (saddle point) $D > 0, f_{xx} < 0$ means concave down all dir. (local max) $D > 0, f_{xx} > 0$ means concave up all dir. (local min)

- **To find critical points:** Find  $f_x(x, y)$  and  $f_y(x, y)$ . Set them BOTH equal to zero, then COMBINE the equations and solve for  $x$  and  $y$ . Check that your points do in fact make both partials equal to zero!
- **To classify critical points:** First find the critical points, then find  $f_{xx}$ ,  $f_{yy}$ , and  $f_{xy}$ . At each critical point compute  $f_{xx}$ ,  $f_{yy}$ ,  $f_{xy}$  and  $D$ . Make appropriate conclusions from the second derivative test.
- **To find absolute max/min over a region:** Draw the region.
  1. Find the critical pts inside the region, label in region.
  2. Over each boundary:
    - (a) Substitution the  $xy$ -equation for that boundary into the surface to get a one variable function and identify the interval.
    - (b) Find the critical numbers and endpoints for this one variable problem (Calculus 1), label these on that boundary.
  3. Plug each point you found into the original function  $z = f(x, y)$ . Biggest output is the max, smallest output is the min.
- **To find a max/min in an applied problem (optimization)**
  1. Draw and label a picture.
  2. *Objective:* What are you optimizing?!? (This is the function you need to find and differentiate!)
  3. *Constraint:* Write down the constraints and use them to make the objective a two-variable function.
  4. Find the critical points of your objective function.
  5. If asked, use the second derivative test to verify that your critical point does indeed give a max or min as desired.