WORKSHEET 2b NOTES AND SOLUTIONS

(1) (SEE PAGE 1 FOR ANSWERS)

NOTE: We defined

TWO LINES ARE PARALLEL $\Leftrightarrow$ THEIR DIRECTION VECTORS ARE PARALLEL

OTHERWISE THEY EITHER INTERSECT OR ARE SKEW.

TWO PLANES ARE PARALLEL $\Leftrightarrow$ THEIR NORMALS ARE PARALLEL

TWO PLANES ARE PERPENDICULAR $\Leftrightarrow$ THEIR NORMALS ARE PERPEN.

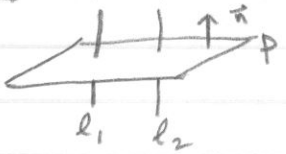
IF TWO PLANES ARE NOT PARALLEL, [illegible]

THEN THEY MUST INTERSECT (IN A LINE).

SPECIFIC COMMENTS:

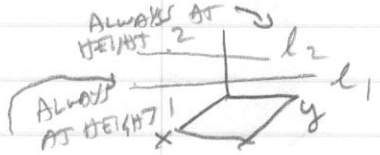
(a) TWO LINES PERPENDICULAR TO SAME PLANE ARE PARALLEL.

TRUE! LINE PERPENDICULAR TO PLANE $\Rightarrow$ LINE DIRECTION VECTOR AND PLANE NORMAL ARE PARALLEL.



(b) TWO LINES PARALLEL TO THE SAME PLANE ARE PARALLEL.

FALSE! THE LINES COULD BE SKEW!



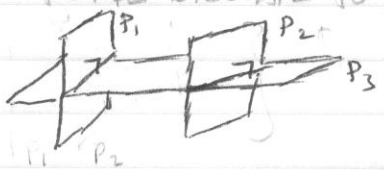
COUNTEREXAMPLE $\longrightarrow$ l_1: $x = 2t, y = t, z = 1$ } BOTH PARALLEL TO xy-PLANE

l_2: $x = 7s, y = 3s, z = 2$ }

AND l_1 & l_2 ARE NOT PARALLEL. (THEY ARE SKEW)

(c) TWO PLANES PERPENDICULAR TO THE SAME PLANE ARE PARALLEL.

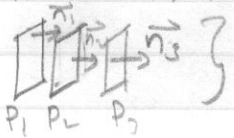
FALSE!



P_1 and P_2 are both perpendicular to P_3, but not to each other. Another example is xy, xz, and yz planes.

(d) TWO PLANES PARALLEL TO THE SAME PLANE ARE PARALLEL

TRUE



$\vec{n}_1$, $\vec{n}_2$, and $\vec{n}_3$ are all parallel

(e) TWO LINES PERPENDICULAR TO THE SAME LINE ARE PARALLEL.

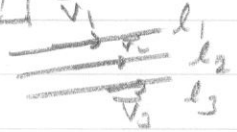
FALSE!

COULD BE SKEW

l_2 AND l_3 DON'T HAVE TO BE PARALLEL

(f) TWO LINES PARALLEL TO THE SAME LINE ARE PARALLEL

TRUE!



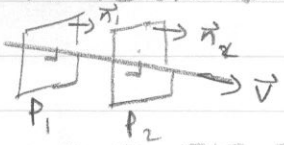
$\vec{v}_1$, $\vec{v}_2$, and $\vec{v}_3$ are all parallel.

(g) TWO PLANES EITHER INTERSECT OR ARE PARALLEL.

TRUE!

(h) TWO PLANES PERPENDICULAR TO THE SAME LINE ARE PARALLEL.

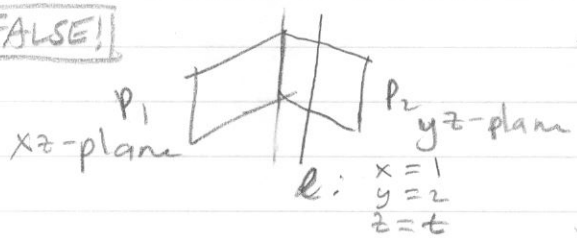
TRUE!



$\vec{n}_1$, $\vec{n}_2$, and $\vec{v}$ are all parallel.

(i) TWO PLANES PARALLEL TO THE SAME LINE ARE PARALLEL.

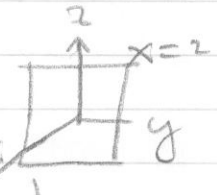
FALSE!



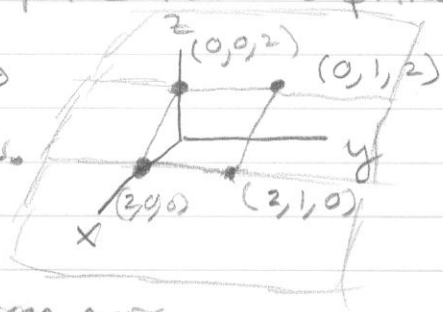
P_1 & P_2 are parallel to ℓ but P_1 and P_2 are not parallel.

(2) PARALLEL TO yz-PLANE AND THROUGH (2,1,3) WOULD BE GIVEN BY $\boxed{x = 2}$

IT IS PERPENDICULAR TO THE xy-plane and xz-plane.



(3) $x + z = 2 \Rightarrow z = 2 - x$ $\longrightarrow$



IT IS NOT PARALLEL TO ANY COORD. PLANES.

IT IS PERPENDICULAR TO THE xz-PLANE.

IT IS PARALLEL TO THE y-AXIS.

IT IS NOT PERPENDICULAR TO ANY COORD. AXIS.

THE NORMAL VECTOR IS $\langle 1, 0, 1 \rangle$!

(Also contains x and z and no y)

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Work with lines and planes.

1. Decide by yourself whether each of the following is true or false. Compare answers with one or two neighbors, then confirm your answers by using pieces of paper and/or a desktop as models for planes, and pens and/or pencils as models for lines.

FOR DETAILS SEE ATTACHED

(a) Two lines perpendicular to the same plane are parallel. TRUE

(b) Two lines parallel to the same plane are parallel. FALSE

(c) Two planes perpendicular to the same (third) plane are parallel. FALSE

(d) Two planes parallel to the same (third) plane are parallel. TRUE

(e) Two lines perpendicular to the same (third) line are parallel. FALSE

(f) Two lines parallel to the same (third) line are parallel. TRUE

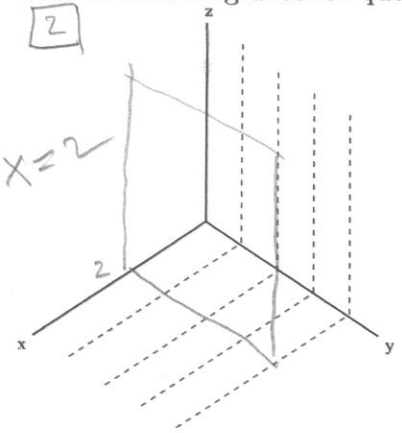
(g) Two planes either intersect or are parallel. TRUE

(h) Two planes perpendicular to the same line are parallel. TRUE

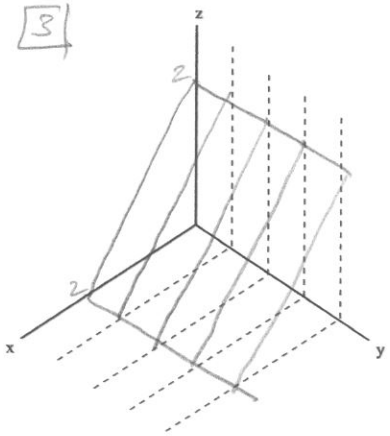
(i) Two planes parallel to the same line are parallel. FALSE

Use the following axes for questions 2 and 3.

2



3



2. Find the equation and sketch the graph of a plane that is parallel to the yz-coordinate plane and contains the point $(2, 1, 3)$. How is this plane related to the other two coordinate planes, the xy-coordiante plane and the xz-coordinate plane?

SEE ATTACHED

3. Graph the plane P given by the equation $x + z = 2$. → NORMAL VECTOR $\langle 1, 0, 1 \rangle$!

SEE ATTACHED

Is P parallel to any of the coordinate planes?
Is P perpendicular to any of the coordinate planes?
Is P parallel to any of the coordinate axes?
Is P perpendicular to any of the coordinate axes?
What fact about the equation for P immediately gives you the anwer to all of these questions?