Worksheet 4: Polar Coordinates

Use this time to discuss your confusions about Polar Coordinates with your TA. You need to be able to plot polar points, convert between Polar and Cartesian, plot enough points to identify a graph, and generally start to learn to think in terms of polar coordinates.

Polar Method: Assume you are standing at the origin facing the positive x-axis. If you rotate counter-clockwise by an angle of θ and walk in the new direction you are facing by r units, then you have reach a point using the polar coordinate method. Conventions:

- Positive θ means rotate counterclockwise and negative θ means rotate clockwise.

- Positive r means walk forward and negative r means walk backward.

You should already know from trig that

$$x = r\cos(\theta) \ , \ y = r\sin(\theta) \ , \ x^2 + y^2 = r^2 \ , \ \tan(\theta) = \frac{y}{x}.$$

1. *Plotting points and regions*

    (a) Sketch the region in the xy-plane such that $1 \le x \le 4$ and $-5 \le y \le -2$.
    (b) Sketch the region in the xy-plane such that $-\frac{\pi}{2} \le \theta \le \pi$ and $2 \le r \le 5$.
    (c) Sketch the region in the xy-plane such that $0 \le \theta \le 2\pi$ and $0 \le r \le 9$.

    Side Note: If you wanted to describe this last example in Cartesian, you would have to write $-3 \le x \le 3, -\sqrt{9 - x^2} \le y \le \sqrt{9 - x^2}$. We will find it is much easier to work with Polar coordinates when dealing with circular regions in chapter 15.

2. *Plotting polar functions*: For all these problems (and the homework), I want you to make a table. Here is how you do it

    - First, plot $\theta = 0, \pi/2, \pi, 3\pi/2$ (*i.e.* plot the first few x- and y-intercepts).
    - Then plot $\theta = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$ (*i.e.* plot 45 degrees into each quadrant).
    - Then plot $\theta = \pi/6, \pi/3, 2\pi/3, 5\pi/6, 7\pi/6, 4\pi/3, 5\pi/3, 11\pi/6$ (*i.e.* plot 30 and 60 degrees into each quadrant)

    You often can identify the general shape (or pick from a list of choices) long before you plot all the points above.

    Try it here, plot several points and connect the dots!

    (a) $r = 1 + \cos(\theta)$ (this is called a Cardiod, it sort of looks like a heart)
    (b) $r = \theta$

Feel free to use the polar graph paper on the next page to help you plot points.