1. (a) (7 pts) A particle is moving according to the position vector function $\mathbf{r}(t) = \langle e^t, 3t, e^{-2t} \rangle$. Find all values of t at which the tangential component of acceleration is zero.

$$a_T = \frac{\vec{r}\,'(t) \cdot \vec{r}\,''(t)}{|\vec{r}\,'(t)|} \stackrel{?}{=} 0 \iff \vec{r}\,'(t) \cdot \vec{r}\,''(t) \stackrel{?}{=} 0$$

$$\vec{r}\,'(t) = \langle e^t, 3, -2e^{-2t} \rangle$$

$$\vec{r}\,''(t) = \langle e^t, 0, 4e^{-2t} \rangle$$

$$\vec{r}\,'(t) \circ \vec{r}\,''(t) = e^{2t} + 0 - 8e^{-4t} \stackrel{?}{=} 0$$

$$e^{2t} = 8e^{-4t}$$

$$e^{6t} = 8$$

$$6t = \ln(8)$$

$$\boxed{t = \tfrac{1}{6}\ln(8)}$$

(b) (7 pts) Find the equation for the tangent plane to $g(x, y) = \dfrac{\sqrt{x^3+1}}{2y} + e^{xy}$ at $(0, 1)$. Then use the tangent plane as a linear approximation to approximate the value of $g(0.1, 0.9)$.

$$g_x(x,y) = \frac{3x^2}{4y\sqrt{x^3+1}} + ye^{xy} \Rightarrow g_x(0,1) = 0 + 1 = 1$$

$$g_y(x,y) = -\frac{\sqrt{x^3+1}}{2y^2} + xe^{xy} \Rightarrow g_y(0,1) = -\tfrac{1}{2} + 0 = -\tfrac{1}{2}$$

$$z_0 = g(0,1) = \frac{1}{2} + 1 = \tfrac{3}{2}$$

TANGENT PLANE: $\boxed{z - \tfrac{3}{2} = 1 \cdot (x - 0) - \tfrac{1}{2}(y - 1)}$

$$g(0.1, 0.9) \approx L(0.1, 0.9) = \frac{3}{2} + (0.1 - 0) - \tfrac{1}{2}(0.9 - 1)$$

$$= 1.5 + 0.1 + 0.05 = \boxed{1.65} = \frac{33}{20}$$

ACTUAL $\approx$ 1.6500075

$x^2y - x^2 - 2y^2$

2. (9 pts) Let $f(x,y) =$ [illegible] $+\, y^3$. Find and classify all critical points of $f(x,y)$. (Classify using appropriate partial derivative tests).

$$f_x(x,y) = 2xy - 2x \stackrel{?}{=} 0 \qquad 2x(y-1) = 0$$

$$x = 0 \quad \text{or } y = 1$$

$$f_y(x,y) = x^2 - 4y \stackrel{?}{=} 0$$

$$x = 0 \Rightarrow 0^2 - 4y = 0 \Rightarrow y = 0 \qquad (0,0)$$

$$y = 1 \Rightarrow x^2 - 4 \stackrel{?}{=} 0 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$$

THREE CRITICAL PTS: $\boxed{(0,0) \quad (-2,1) \quad (2,1)}$

SECOND DERIVATIVE TEST

$$f_{xx}(x,y) = 2y - 2, \quad f_{yy}(x,y) = -4, \quad f_{xy}(x,y) = 2x$$

$$D(x,y) = (2y-2)(-4) - (2x)^2$$

$$(0,0) \Rightarrow D(0,0) = (-2)(-4) - 0 = 8 > 0$$
$$f_{xx}(0,0) = -2 < 0$$
} LOCAL MAX

$$(-2,1) \Rightarrow D(-2,1) = (0)(-4) - (-4)^2 = -16 < 0$$
$$(2,1) \Rightarrow D(2,1) = (0)(-4) - (4)^2 = -16 < 0$$
} SADDLE POINTS

3. (a) (7 pts) Set up and evaluate a double integral to find the volume of the solid below the surface $z + 3x^2 - 5y^2 = 12$ and bounded by the planes, $x = 0$, $x = 2$, $y = 0$, $y = 3$ and $z = 0$.

$$\int_0^3 \int_0^2 12 + 5y^2 - 3x^2 \, dx\, dy$$

$$\int_0^3 12x + 5y^2 x - x^3 \Big|_0^2 \, dy$$

$$\int_0^3 24 + 10y^2 - 8 \, dy$$

$$\int_0^3 16 + 10y^2 \, dy$$

$$16y + \frac{10}{3} y^3 \Big|_0^3$$

$$16 \cdot 3 + 90 = \boxed{138}$$

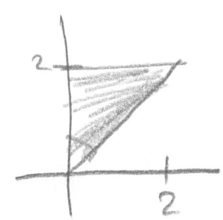
(b) (7 pts) Evaluate the integral by reversing the order of integration: $\displaystyle\int_0^2 \int_x^2 e^{y^2} \, dy dx$.

$0 \le x \le 2$

$x \le y \le 2$

$0 \le y \le 2$

$0 \le x \le y$



$$\int_0^2 \int_0^y e^{y^2} \, dx\, dy$$

$$\int_0^2 x e^{y^2} \Big|_0^y \, dy$$

$$\int_0^2 y e^{y^2} \, dy$$

$u = y^2$

$du = 2y\, dy$

$$\int_0^4 e^u \frac{1}{2} \, du$$

$$\frac{1}{2} e^u \Big|_0^4 = \frac{1}{2} e^4 - \frac{1}{2} e^0 = \boxed{\frac{1}{2}(e^4 - 1)}$$

4. (13 pts) For both questions below, consider the region $D = \{(x,y) \mid x \le 0,\ y \ge 0,\ x^2 + y^2 \le 9\}$.

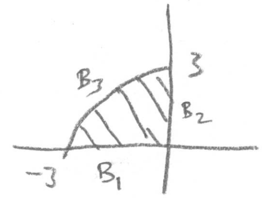
(a) (7 pts) Find the absolute maximum and absolute minimum of $f(x,y) = yx^2 + 10$ over D.

$f_x(x,y) = 2yx \overset{?}{=} 0$ $\quad$ $y =$ anything

$f_y(x,y) = x^2 \overset{?}{=} 0 \Rightarrow x = 0$

CRITICAL PTS

$(0, y) \to$ ALL OF B_2



Boundary

$\boxed{B_1}$ $y = 0,\ -3 \le x \le 0 \Rightarrow z = f(x,0) = 10$ $\leftarrow$ a constant

$\boxed{B_2}$ $x = 0,\ 0 \le y \le 3 \Rightarrow z = f(0,y) = 10$ $\leftarrow$ a constant

$\boxed{B_3}$ $x = -\sqrt{9-y^2},\ 0 \le y \le 3 \Rightarrow z = f(-\sqrt{9-y^2}, y) = y(9-y^2) + 10$

$z = 9y - y^3 + 10$

$\frac{dz}{dy} = 9 - 3y^2 \overset{?}{=} 0 \Rightarrow y^2 = 3$

$y = \pm\sqrt{3}$

not in region

$y = \sqrt{3} \Rightarrow x = -\sqrt{9-3} = -\sqrt{6}$

$f(-\sqrt{6}, \sqrt{3}) = \sqrt{3} \cdot 6 + 10 = 10 + 6\sqrt{3}$

$$\boxed{\begin{array}{l} \text{ABSOLUTE MINIMUM} = 10 \\ \text{ABSOLUTE MAXIMUM} = 10 + 6\sqrt{3} \end{array}} \approx 20.3923$$

(b) (6 pts) Using polar coordinates, evaluate: $\displaystyle\iint_D y + \sqrt{x^2+y^2}\, dA$.

$\pi/2 \le \theta \le \pi$

$0 \le r \le 3$

$$\int_{\pi/2}^{\pi} \int_0^3 (r\sin(\theta) + r)\, r\, dr\, d\theta$$

$$\int_{\pi/2}^{\pi} (\sin(\theta)+1) \int_0^3 r^2\, dr\, d\theta = \int_{\pi/2}^{\pi} (\sin(\theta)+1) \underbrace{\tfrac{1}{3} r^3 \Big|_0^3}_{=9} d\theta$$

$$9 \int_{\pi/2}^{\pi} \sin(\theta) + 1\, d\theta = 9\left(-\cos(\theta) + \theta \Big|_{\pi/2}^{\pi}\right)$$

$$= 9\left[(-(-1) + \pi) - (0 + \pi/2)\right]$$

$$= \boxed{9(1 + \pi/2)} = 9 + \tfrac{9}{2}\pi$$

≈ 23.1372