**1** (8 points) Let $\mathbf{r}(t) = (2t-1)\,\mathbf{i} + t^2\,\mathbf{j} + 2\sqrt{t}\,\mathbf{k}$. Find all times $t$ when the tangential component of acceleration is zero.

*The tangential component of acceleration at time $t$ is* $a_T = \dfrac{\mathbf{r}'(t)\cdot\mathbf{r}''(t)}{|\mathbf{r}'(t)|}$.

*Thus* $a_T = 0$ *when* $\mathbf{r}'(t)\cdot\mathbf{r}''(t) = 0$.

$$\begin{aligned}\mathbf{r}'(t)\cdot\mathbf{r}''(t) &= \left\langle 2, 2t, t^{-1/2}\right\rangle \cdot \left\langle 0, 2, -\frac{1}{2}t^{-3/2}\right\rangle \\ &= 4t - \frac{1}{2}t^{-2}\end{aligned}$$

*Solving* $4t - \frac{1}{2}t^{-2} = 0$ *gives* $8t^3 - 1 = 0$ *so* $t = \frac{1}{2}$ *is the only solution.*

**2** (6 points) Find the equation of the tangent plane of the function $F(x,y) = \dfrac{3y-2}{5x+7}$ at the point $(-1, 1)$.

$F_x(x,y) = -5 \cdot \dfrac{3y-2}{(5x+7)^2}$ *so* $F_x(-1,1) = -\dfrac{5}{4}$

$F_y(x,y) = \dfrac{3}{5x+7}$ *so* $F_y(-1,1) = \dfrac{3}{2}$

$F(-1,1) = \dfrac{1}{2}$

*The equation of the tangent plane is* $z - \dfrac{1}{2} = -\dfrac{5}{4}(x+1) + \dfrac{3}{2}(y-1)$

3 (14 points) Evaluate the following double integrals.

(a) (7 points) $\iint_R xy\sin(x^2y)\,dA, \quad R = [0,1]\times[0,\pi/2]$

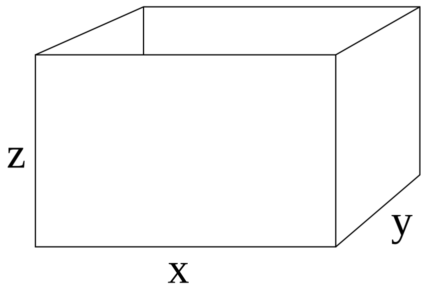
*Use Fubini's theorem to conver this to an iterated integral.*

$$\begin{aligned}\iint_R xy\sin(x^2y)\,dA &= \int_0^{\pi/2}\int_0^1 xy\sin(x^2y)\,dx\,dy\\ &= \int_0^{\pi/2}\left(-\frac{1}{2}\cos(x^2y)\Big|_{x=0}^{1}\right)dy\\ &= \int_0^{\pi/2}\frac{1}{2}-\frac{1}{2}\cos(y)\,dy\\ &= \frac{1}{2}y-\frac{1}{2}\sin(y)\Big|_{y=0}^{\pi/2}\\ &= \frac{\pi}{4}-\frac{1}{2}\end{aligned}$$

(b) (7 points) $\iint_D y^2e^{xy}\,dA, \quad D = \{\ (x,y)\mid 0\le y\le 3,\ 0\le x\le y\ \}$

$$\begin{aligned}\iint_D y^2e^{xy}\,dA &= \int_0^3\int_0^y y^2e^{xy}\,dx\,dy\\ &= \int_0^3\left(ye^{xy}\Big|_{x=0}^{y}\right)dy\\ &= \int_0^3 ye^{y^2}-y\,dy\\ &= \frac{1}{2}e^{y^2}-\frac{1}{2}y^2\Big|_{y=0}^{3}\\ &= \frac{1}{2}e^9-5\end{aligned}$$

**4** (12 points) You wish to build a rectangular box with no top with volume 6 ft$^3$. The material for the bottom is metal and costs \$3.00 a square foot. The sides are wooden and cost \$2.00 a square foot. Calculate the dimesnsions of the box with minimum cost. Use the Second Derivative test to verify that your answer is indeed a minimum.




*Label the sides as shown. The cost is* $C = 3xy + 4xz + 4yz$. *The volume constraint is* $xyz = 6$.

*From the constraint we get* $z = \dfrac{6}{xy}$. *Substituting into the cost function gives the objective function* $C(x, y) = 3xy + \dfrac{24}{y} + \dfrac{24}{x}$.

*Now calculate the partial derivatives and set them equal to zero.*

$C_x(x, y) = 3y - \dfrac{24}{x^2} = 0$ *gives* $y = \dfrac{8}{x^2}$.

$C_y(x, y) = 3x - \dfrac{24}{y^2} = 0$ *gives* $x = \dfrac{8}{y^2}$.

*Combining, we get* $x = \dfrac{8}{(8/x^2)^2}$ *or* $8x = x^4$. *The solutions are* $x = 0, 2$ *but* $x = 0$ *makes no sense and can be discarded. If* $x = 2$ *and* $y = \dfrac{8}{x^2}$ *we get* $y = 2$ *as well. Since* $xyz = 6$ *it follows that* $z = \dfrac{3}{2}$.

*The dimensions of the box are* $2 \times 2 \times \dfrac{3}{2}$.

*To verify that this is gives the minimum cost, we must compute the second derivatives.*

$C_{xx}(2, 2) = \left.\dfrac{48}{x^3}\right|_{x=2} = 6$

$C_{yy}(2, 2) = \left.\dfrac{48}{y^3}\right|_{y=2} = 6$

$C_{xy}(x, y) = 3$

*Thus the Hessian determinant is* $6 \cdot 6 - 3 \cdot 3 > 0$. *Since* $C_{xx}(2, 2) > 0$, *the point* $(2, 2)$ *gives a minimum of the cost function by the Second Derivative Test.*

**5** (10 points) A table of values is given for a function $g(x,y)$ defined on $R = [0,1] \times [1,4]$. (For example, $g(1,4) = 9.4$.) Use the table to find a linear approximation to $g(x,y)$ near $(0.5,3)$. Use it to approximate $g(0.6,2.8)$. Carefully explain all your reasoning.

| | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 |
|---|---|---|---|---|---|---|---|
| 0 | 1 | 1.8 | 2.8 | 3.9 | 5.2 | 6.5 | 8.0 |
| 0.25 | 1.2 | 1.9 | 2.9 | 4.0 | 5.3 | 6.6 | 8.2 |
| 0.5 | 1.4 | 2.1 | 3.1 | 4.2 | 5.5 | 6.8 | 8.5 |
| 0.75 | 1.6 | 2.2 | 3.3 | 4.5 | 5.8 | 7.0 | 8.9 |
| 1 | 1.7 | 2.3 | 3.6 | 4.8 | 6.1 | 7.3 | 9.4 |

*We need to approximate the partial derivatives* $g_x(0.5,3)$ *and* $g_y(0.5,3)$. *There are several correct ways to do this. I will choose one.*

*I approximate* $g_x(0.5,3)$ *with the slope of the secant line from* $(0.5,3,5.5)$ *to* $(0.75,3,5.8)$.

*The slope is* $\dfrac{\Delta z}{\Delta x} = \dfrac{5.8-5.5}{0.75-0.5} = 1.2$.

*I approximate* $g_y(0.5,3)$ *with the slope of the secant line from* $(0.5,3,5.5)$ *to* $(0.5,2.5,4.2)$.

*The slope is* $\dfrac{\Delta z}{\Delta y} = \dfrac{4.2-5.5}{2.5-3} = 2.6$.

*The linear approximation* $L(x,y) = 5.5 + 1.2(x-0.5) + 2.6(y-3)$.

$L(0.6,2.8) = 5.5 + 1.2 \cdot 0.1 - 2.6 \cdot 0.2 = 5.1$