Exam 2 Facts

Slopes on Surfaces.

Be able to find and graph the domain	Know the basics on level curves/contour maps
$f_x(x,y) = \frac{\partial z}{\partial x}$ = slope in x-direction	$f_y(x,y) = \frac{\partial z}{\partial y}$ = slope in y-direction
$z = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$	Tangent plane/linearization.
$f_{xx}(x,y) = \frac{\partial^2 z}{\partial x^2}$ = concavity in x-direction	$f_{yy}(x,y) = \frac{\partial^2 z}{\partial y^2}$ = concavity in y-direction
$f_{xy}(x,y) = \frac{\partial^2 z}{\partial y \partial x}$ = mixed second partial	$f_{xy}(x,y) = f_{yx}(x,y)$ (Clairaut's Theorem)
$D = f_{xx} \cdot f_{yy} - (f_{xy})^2$ = measure of concavity	$D < 0$ means concavity changes (saddle)
$D > 0$, $f_{xx} > 0$ means concave up all directions	$D > 0$, $f_{xx} < 0$ means concave down all directions

Comments:

- To find critical points: Find f_x and f_y, set them BOTH equal to zero, then COMBINE the equations and solve for x and y.
- To classify critical points: Find f_{xx}, f_{yy}, and f_{xy}. At each critical point compute f_{xx}, f_{yy}, f_{xy} and D and make appropriate conclusions from the second derivative test.
- To find absolute max/min on a region: Find critical points inside the region. Then, over each boundary, substitution the xy-equation for the boundary into the surface to get a one variable function. Find the absolue max/min of the one variable function over each boundary. In the end, evaluate $f(x,y)$ at all the critical points inside the region and the critical numbers and endpoints (corners) on each boundary to find the largest and smallest output.

Volumes: $\iint\limits_D f(x,y)\,dA$ = signed volume 'above' the xy-axis, 'below' $f(x,y)$ and inside the region D.

We also saw $\iint\limits_D 1\,dA$ = area of D.

To set up a double integral:
(1) Solve for integrand ($z = f(x,y)$).
(2) Draw given xy-equations in the xy-plane (label intersections).
(3) Draw xy-equations that occur from surface intersections.
(4) Set up the double integral(s) using the region for D.

Options for set up:

$\iint\limits_D f(x,y)\,dA = \int_a^b \int_{g(x)}^{h(x)} f(x,y)\,dydx$,	$y = g(x)$ = bottom,	$y = h(x)$ = top
$\iint\limits_D f(x,y)\,dA = \int_c^d \int_{p(y)}^{q(y)} f(x,y)\,dxdy$,	$x = p(y)$ = left,	$x = q(y)$ = right
$\iint\limits_D f(x,y)\,dA = \int_\alpha^\beta \int_{w(\theta)}^{v(\theta)} f(r\cos(\theta), r\sin(\theta))\,rdrd\theta$,	$r = w(\theta)$ = inner,	$r = v(\theta)$ = outer

Center of Mass Application: If $\rho(x,y)$ = formula for density at a point in the region D, then

$$M = \text{total mass} = \iint\limits_D \rho(x,y)\ dA \ , \quad \overline{x} = \frac{\iint\limits_D x\rho(x,y)\ dA}{\iint\limits_D \rho(x,y)\ dA} \quad \text{and} \quad \overline{y} = \frac{\iint\limits_D y\rho(x,y)\ dA}{\iint\limits_D \rho(x,y)\ dA}$$