Exam 1 Review Overheads

Exam 1 details:

- 4 pages of questions
- ONLY the Ti-30x IIS Calculator model is allowed (you will want this!)
- Allowed one **hand-written** 8.5 by 11 inch page of notes (double-sided)
- You must show your work on all problems.
- Covers 12.1-12.6, 13.1-13.4. You should know all the facts and concepts covered in lecture and in homework for those sections.
- You have 50 minutes to complete the exam.

Studying Advice:

- Spend 15-30 minutes reviewing all homework.
- Spend 15-30 minutes flipping through several old exams.
- Spend several hours working through several old exams in detail.
- Practice managing your time, never spend more than 10 minutes on a page!

Exam 1 Basic Facts

1. Vector Operations: Sums, scalar multiples, dot products, cross products.
2. Vector Facts: checking orthogonality, checking parallel, angle between, area of parallelogram/triangle, projections.
3. Finding Line and Plane Equations.
4. Knowing basics of traces and knowing the 7 basic shapes and their names.
5. Working with Parametric Equations in $\mathbf{R}^3$:
   Tangent vector, unit tangent, tangent line, unit normal, arc length, curvature, velocity, acceleration.

Basic Vector Facts:

1. $\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$.
2. $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$.
3. $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}||\mathbf{v}|\cos(\theta)$.
4. $\mathbf{u} \cdot \mathbf{v} = 0$ means orthogonal.
5. $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}||\mathbf{v}|\sin(\theta)$.
6. $\mathbf{u} \times \mathbf{v}$ is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$.
7. $|\mathbf{u} \times \mathbf{v}| =$ parallelogram area.
8. $\text{comp}_{\mathbf{a}}(\mathbf{b}) = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|}$.
9. $\mathbf{proj}_{\mathbf{a}}(\mathbf{b}) = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2}\mathbf{a}$.

Basic Lines, Planes and Surfaces:

1. Lines: $x = x_0 + at$, $y = y_0 + bt$, $z = z_0 + ct$
   $(x_0, y_0, z_0) =$ a point on the line
   $\langle a, b, c \rangle =$ a direction vector
2. Planes: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$
   $(x_0, y_0, z_0) =$ a point on the plane
   $\langle a, b, c \rangle =$ a normal vector
3. Cylinder: One variable ‘missing’
   (Assume a,b,c positive below)
4. Elliptical/Circular Paraboloid: $z = ax^2 + by^2$
5. Hyperbolic Paraboloid: $z = ax^2 - by^2$.
6. Ellipsoid/Sphere: $ax^2 + by^2 + cz^2 = 1$.
7. Elliptical/Circular Cone: $z^2 = ax^2 + by^2$.
8. Hyperboloid of One Sheet: $ax^2 + by^2 - cz^2 = 1$.
9. Hyperboloid of Two Sheets: $ax^2 + by^2 - cz^2 = -1$.

Basic Parametric in R^3:

1. $\mathbf{r}'(t) = \langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \rangle$.

2. $\mathbf{r}''(t) = \langle \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2z}{dt^2} \rangle$.

3. $\displaystyle\int \mathbf{r}(t)\, dt = \left\langle \int x(t)dt, \int y(t)dt, \int z(t)dt \right\rangle$.

4. $\mathbf{T}(t) = \frac{1}{|\mathbf{r}'(t)|}\mathbf{r}'(t)$.

5. $\mathbf{N}(t) = \frac{1}{|\mathbf{T}'(t)|}\mathbf{T}'(t) =$ principal unit normal

6. $s =$ Arc Length $= \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}\, dt$.

7. $\kappa(t) = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$.

8. For a function $y = f(x)$ in R^2, the curvature formula simplifies to $\kappa(x) = \frac{|f''(x)|}{(1+(f'(x))^2)^{3/2}}$.

9. $\mathbf{r}'(t) = \mathbf{v}(t) =$ velocity vector

10. $|\mathbf{r}'(t)| = |\mathbf{v}(t)| =$ speed

11. $\mathbf{r}''(t) = \mathbf{a}(t) =$ acceleration

12. $a_T = \frac{\mathbf{r}'(t)\cdot\mathbf{r}''(t)}{|\mathbf{r}'(t)|}$

13. $a_N = \frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|}$