13.2 Calculus for Vector Curves

Now we are embarking on calculus in 3 dimensions. Ultimately, you will see that all you have to do is to differentiate and integrate **component-wise**. We will be using and interpreting these vectors in lecture and homework. Here are the basic calculus definitions of this section:

Derivatives

If $\mathbf{r}(t) = \langle x(t), y(t), z(t)\rangle$, then

$$\mathbf{r}'(t) = \langle x'(t), y'(t), z'(t)\rangle \qquad \text{a vector tangent to the curve at } t.$$

Sometimes, we want the tangent vector to have length one. In which case, we compute

$$\mathbf{T}(t) = \frac{1}{|\mathbf{r}'(t)|}\mathbf{r}'(t) \qquad \text{the unit tangent vector.}$$

Check out the vector derivative rules in your text. In particular, notice that their are three types of product rules (scalar function product rule, dot product rule, and cross product rule). These rules should look familiar.

The second derivative is defined by:

$$\mathbf{r}''(t) = \langle x''(t), y''(t), z''(t)\rangle.$$

Integrals

If $r(t) = \langle x(t), y(t), z(t)\rangle$, then

$$\int \mathbf{r}(t)dt = \left\langle \int x(t)dt, \int y(t)dt, \int z(t)dt \right\rangle,$$

where each has its own different constant of integration.

Notes:

1. The **equation for the tangent line** to $(x(t), y(t), z(t))$ at $t = a$ is given by

$$\begin{aligned} x &= x(a) + x'(a)u \\ y &= y(a) + y'(a)u \\ z &= z(a) + z'(a)u \end{aligned}$$

   Note that $\mathbf{r}'(a) = \langle x'(a), y'(a), z'(a)\rangle$ is a direction vector for the tangent line.

2. If you need the angle of intersection of two curves, then you need the angle between their tangent vectors at their intersection.