1. A particle is moving in such a way that it's acceleration is given by $\mathbf{a}(t) = \langle 0, e^t, 6\cos(2t)\rangle$. The initial velocity is $\mathbf{v}(0) = \langle 0, 3, 0\rangle$ and the initial position is $\mathbf{r}(0) = \langle 1, 0, 1\rangle$.

(a) (6 pts) Find the position vector, $\mathbf{r}(t)$.

$\vec{v}(t) = \langle c_1, e^t + c_2, 3\sin(2t) + c_3\rangle$

$\vec{v}(0) = \langle 0, 3, 0\rangle \Rightarrow c_1 = 0, \; 1 + c_2 = 3, \; 0 + c_3 = 0$

$\vec{v}(t) = \langle 0, e^t + 2, 3\sin(2t)\rangle$

$\vec{r}(t) = \langle d_1, e^t + 2t + d_2, -\frac{3}{2}\cos(2t) + d_3\rangle$

$\vec{r}(0) = \langle 1, 0, 1\rangle \Rightarrow d_1 = 1, \; 1 + 0 + d_2 = 0, \; -\frac{3}{2} + d_3 = 1$

$d_2 = -1 \qquad d_3 = 5/2$

$$\boxed{\vec{r}(t) = \langle 1, e^t + 2t - 1, -\tfrac{3}{2}\cos(2t) + 5/2\rangle}$$

(b) (4 pts) Find the normal component of acceleration of $\mathbf{r}(t)$ at $t = 0$.

$a_N = \dfrac{|\vec{r}\,'(0) \times \vec{r}\,''(0)|}{|\vec{r}\,'(0)|}$

$\vec{r}\,'(0) = \vec{v}(0) = \langle 0, 3, 0\rangle$

$\vec{r}\,''(0) = \vec{a}(0) = \langle 0, 1, 6\rangle$

$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 3 & 0 \\ 0 & 1 & 6 \end{vmatrix} = \langle 18, 0, 0\rangle$

$$\boxed{a_N = \frac{18}{3} = 6}$$

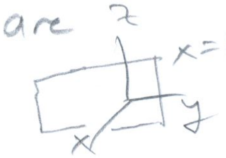
(c) (2 pts) For the curve $\mathbf{r}(t)$, the binormal, $\mathbf{B}(t)$, is always ~~pointing in the direction of one~~ parallel to one of the axes. (Circle the correct one).

i. $\mathbf{B}(t)$ is always parallel to the x-axis.

ii. $\mathbf{B}(t)$ is always parallel to the y-axis.

iii. $\mathbf{B}(t)$ is always parallel to the z-axis.

SINCE $x = 1$ IS CONSTANT, ALL MOTION IS ON THIS PLANE. THUS, $\vec{T}$, $\vec{N}$, $\vec{r}\,'$, and $\vec{r}\,''$ are all on the plane $x = 1$. SO $\vec{B}$ IS parallel to the X-axis



2. (a) Consider the surface defined implicitly by the equation $xz^3 = 8(\sin(xy)+1)$.

i. (5 pts) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at the point $(1, 0, 2)$.

$$\frac{\partial}{\partial x}: \quad z^3 + 3xz^2\frac{\partial z}{\partial x} = 8y\cos(xy) \Rightarrow \frac{\partial z}{\partial x} = \frac{8y\cos(xy) - z^3}{3xz^2}$$

$$\frac{\partial}{\partial y}: \quad 3xz^2\frac{\partial z}{\partial y} = 8x\cos(xy) \Rightarrow \frac{\partial z}{\partial y} = \frac{8x\cos(xy)}{3xz^2}$$

AT $(1,0,2)$

$$\boxed{\frac{\partial z}{\partial x} = \frac{0-8}{3(1)(4)} = -\frac{8}{12} = -\frac{2}{3}}, \quad \boxed{\frac{\partial z}{\partial y} = \frac{8}{3(1)(2)^2} = \frac{2}{3}}$$

ii. (3 pts) Use the linear approximation at $(1, 0, 2)$, to estimate the z-value on the surface that corresponds to $x = 1.1$ and $y = -0.2$.

$$z = 2 - \tfrac{2}{3}(x-1) + \tfrac{2}{3}(y-0) \quad \text{at } (1.1, -0.2, ?)$$

$$z = 2 - \tfrac{2}{3}(1.1-1) + \tfrac{2}{3}(-0.2) = 2 - \tfrac{2}{3}(0.1) - \tfrac{2}{3}(0.2)$$

$$= 2 - \tfrac{2}{3}\tfrac{3}{10} = \boxed{1.8}$$

ACTUAL =

(b) (6 pts) Consider the region of integration for the double integral that looks like:

$$\int_0^3 \int_{(y-1)/2}^{1} f(x,y)\,dx\,dy.$$

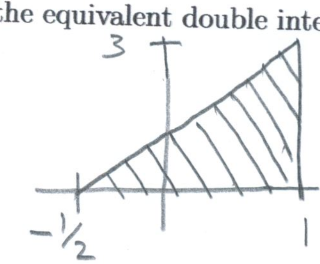
Draw the region of integration. And give the equivalent double integral in the reverse order.

RIGHT : $x = 1$

LEFT : $x = (y-1)/2$

$2x = y - 1$

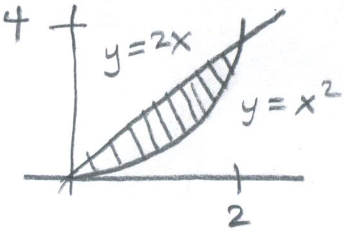
$y = 2x + 1$



$$\boxed{\int_{-1/2}^{1} \int_0^{2x+1} f(x,y)\,dy\,dx}$$

3. (a) (6 pts) Find the volume of the solid bounded by the surfaces $y = 2x$, $y = x^2$, $z = 0$, and $z = 6y$.

$$\text{VOLUME} = \int_0^2 \int_{x^2}^{2x} 6y\,dy\,dx \overset{\text{OR}}{=} \int_0^4 \int_{y/2}^{\sqrt{y}} 6y\,dydx$$



$$= \int_0^2 3y^2 \Big|_{x^2}^{2x} dx$$

$$= \int_0^2 3(2x)^2 - 3(x^2)^2 dx$$

$$= \int_0^2 12x^2 - 3x^4 dx$$

$$= 4x^3 - \tfrac{3}{5}x^5 \Big|_0^2$$

$$= 4(2)^3 - \tfrac{3}{5}(2)^5$$

$$= 32 - \tfrac{3}{5}32 = \boxed{\tfrac{64}{5} = 12.8}$$

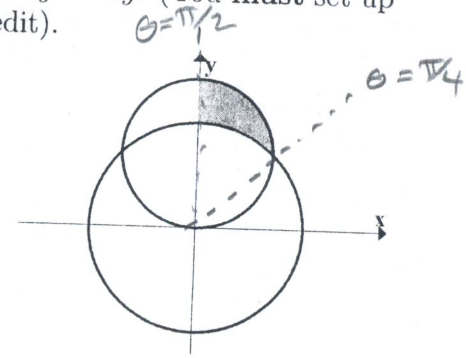
(b) (7 pts) Using a double integral in polar coordinates, find the area of the region in the first quadrant that is outside of $x^2 + y^2 = 2$ and inside the circle $x^2 + y^2 = 2y$. (You **must** set up and evaluate a double integral in polar coordinates for full credit).

$$x^2+y^2=2 \Rightarrow r^2=2 \Rightarrow r=\sqrt{2}$$

$$x^2+y^2=2y \Rightarrow r^2=2r\sin\theta \Rightarrow r=2\sin\theta$$

INTERSECTION: $2\sin\theta=\sqrt{2} \Rightarrow \sin\theta = \frac{\sqrt{2}}{2}$

$\Rightarrow \theta = \pi/4$



$$\iint_R 1\,dA = \int_{\pi/4}^{\pi/2} \int_{\sqrt{2}}^{2\sin\theta} 1\, r\,dr\,d\theta$$

$$= \int_{\pi/4}^{\pi/2} \tfrac{1}{2} r^2 \Big|_{\sqrt{2}}^{2\sin\theta} d\theta$$

$$= \int_{\pi/4}^{\pi/2} 2\sin^2\theta - 1\, d\theta$$

$$= \int_{\pi/4}^{\pi/2} \cancel{1} - \cos(2\theta) - \cancel{1}\, d\theta$$

$$= -\tfrac{1}{2}\sin(2\theta) \Big|_{\pi/4}^{\pi/2}$$

$$= (-0) - (-\tfrac{1}{2}) = \boxed{\tfrac{1}{2}}$$

4. Let $z = f(x, y) = x^2 + 4y - x^2y + 1$. Use this function to answer both parts below.

(a) (5 pts) Find and classify all critical points. (Show your use of the 2nd derivative test).

$$f_x(x,y) = 2x - 2xy \overset{?}{=} 0 \qquad \begin{cases} x=-2 \Rightarrow 2(-2)-2(-2)y=0 \Rightarrow y=1 \\ x=2 \Rightarrow 2(2)-2(2)y=0 \Rightarrow y=1 \end{cases}$$

$$f_y(x,y) = 4 - x^2 \overset{?}{=} 0 \Rightarrow x = \pm 2$$

$$\boxed{\text{CRITICAL POINTS: } (-2,1),\ (2,1)}$$

$f_{xx} = 2-2y$

$f_{yy} = 0$

$f_{xy} = -2x$

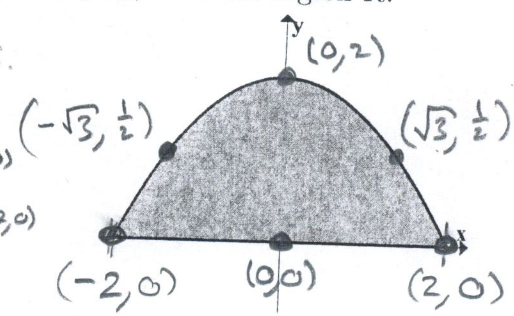
$D = -(-2x)^2$

$(-2,1)$, $(2,1)$ $\Rightarrow D = -(-2(\pm 2))^2 = -16 < 0$

$$\boxed{\text{BOTH SADDLE POINTS}}$$

(b) (6 pts) Let R be the region in the xy-plane consisting of all points bounded by $y = 2 - \frac{1}{2}x^2$ and the x-axis. Find the global minimum and maximum of $f(x, y)$ over the region R.

CRITICAL POINTS ARE NOT IN REGION.
SO WE JUST NEED TO CONSIDER THE BOUNDARIES.



$\boxed{\text{I}}$ $y=0 \Rightarrow z = x^2+1$
$z' = 2x \overset{?}{=} 0$

CRITICAL NUMBERS $x=0 \Rightarrow (0,0)$
ENDPOINTS $x = \pm 2 \Rightarrow (\pm 2, 0)$

$\boxed{\text{II}}$ $y = 2 - \frac{1}{2}x^2 \Rightarrow z = x^2 + 4(2-\frac{1}{2}x^2) - x^2(2 - \frac{1}{2}x^2) + 1$

$z = \frac{1}{2}x^4 - 3x^2 + 9$

$z' = 2x^3 - 6x \overset{?}{=} 0$

$2x(x^2-3) \overset{?}{=} 0$

CRITICAL NUMBERS $x=0 \Rightarrow (0,2)$
$x = \pm\sqrt{3} \Rightarrow (\pm\sqrt{3}, \frac{1}{2})$
ENDPOINTS $x = \pm 2 \Rightarrow (\pm 2, 0)$

YOU <u>MUST</u> FIND ALL SIX OF THESE POINTS FOR FULL CREDIT.

$f(\pm 2, 0) = (\pm 2)^2 + 1 = 5$

$f(0,0) = 0^2 + 1 = 1$

$f(\pm\sqrt{3}, \frac{1}{2}) = \frac{9}{2} - 9 + 9 = 4.5$

$f(0,2) = 0 - 0 + 9 = 9$

$$\boxed{\begin{array}{l} \text{ABS MIN} = 1 \\ \text{ABS MAX} = 9 \end{array}}$$