1. (12 points)

(a) (4 pts) Find the equation of the plane that contains the point $(7, -5, 6)$ and is orthogonal to the line given by the symmetric equations $\frac{x-1}{2} = \frac{y-2}{3} = \underbrace{-z}$.

EQUIVALENT TO $\frac{z-0}{-1}$

LINE DIRECTION VECTOR $= \langle 2, 3, -1 \rangle$

THIS VECTOR IS ORTHOGONAL TO THE DESIRED PLANE, WE CAN USE IT AS A NORMAL VECTOR.

$$\boxed{2(x-7)+3(y+5)-(z-6)=0}$$

(b) Consider the surface $z = x^2 - 3y^2$.

i. (3 pts) Name the surface. That is, state whether this equation gives a cone, an ellipsoid, a parabolic cylinder, or one of the other named shapes that we discussed in section 12.6 (if you can't think of the name, describe all the traces for partial credit).

$\boxed{\text{HYPERBOLIC PARABOLOID}}$

TRACES $\begin{cases} \text{HYPERBOLAS} \leftarrow xy \\ \text{PARABOLAS} \leftarrow xz \\ \text{PARABOLAS} \leftarrow yz \end{cases}$

ii. (5 pts) Find **all** (x, y, z) intersection points of the line through $(0, 0, 9)$ and $(2, 1, 9)$ with the surface $z = x^2 - 3y^2$. (Hint: Start by finding equations for the line).

LINE: DIRECTION VECTOR $= \langle 2-0, 1-0, 9-9 \rangle = \langle 2, 1, 0 \rangle$

$x = 0 + 2t, \; y = 0 + t, \; z = 9$

INTERSECTION: $z = x^2 - 3y^2$

$$9 \stackrel{?}{=} (2t)^2 - 3(t)^2$$

$$\Rightarrow 9 = 4t^2 - 3t^2$$

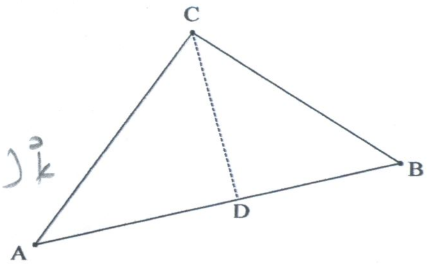
$$\Rightarrow 9 = t^2 \Rightarrow t = \pm 3$$

POINTS: $t = -3 \Rightarrow x = 2(-3), y = -3, z = 9 \Rightarrow (x, y, z) = (-6, -3, 9)$

$t = 3 \Rightarrow x = 2(3), y = (3), z = 9 \rightarrow (x, y, z) = (6, 3, 9)$

2. (14 pts) Consider the triangle shown. The coordinates for C are (4,3,1).
You are given $\overrightarrow{BA} = \langle -2, -1, 3\rangle$ and $\overrightarrow{BC} = \langle -1, 1, 4\rangle$.
The dotted line CD is perpendicular to BA.
Answer the following questions (Leave your answer in exact form, you do not have to simplify).

(a) (5 pts) Find the equation of the plane that contains the points A, B, and C.



$$\vec{n} = \overrightarrow{BA} \times \overrightarrow{BC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & -1 & 3 \\ -1 & 1 & 4 \end{vmatrix}$$

$$= (-4 - 3)\vec{i} - (-8 - -3)\vec{j} + (-2 - 1)\vec{k}$$

$$= \langle -7, 5, -3 \rangle$$

$$\boxed{-7(x-4) + 5(y-3) - 3(z-1) = 0}$$

(b) (3 pts) Find the area of the triangle ABC.

$$\text{AREA} = \frac{1}{2}\left|\overrightarrow{BA} \times \overrightarrow{BC}\right| = \boxed{\frac{1}{2}\sqrt{(-7)^2 + (5)^2 + (3)^2} = \frac{1}{2}\sqrt{49+25+9} = \frac{1}{2}\sqrt{83}}$$

(c) (3 pts) Find the coordinates for the point A.

$\overrightarrow{BC} = \langle -1, 1, 4\rangle$ and $C = (4,3,1) \Rightarrow B = (4 - -1, 3 - 1, 1 - 4)$

$B = (5, 2, -3)$

$\overrightarrow{BA} = \langle -2, -1, 3\rangle$ and $B = (5, 2, -3) \Rightarrow A = (5 + -2, 2 + -1, -3 + 3)$

$\boxed{A = (3, 1, 0)}$

(d) (3 pts) Find the distance from B to D.

$$|BD| = \text{comp}_{\overrightarrow{BA}} \overrightarrow{BC} = \frac{\overrightarrow{BC} \cdot \overrightarrow{BA}}{|\overrightarrow{BA}|} = \frac{(-1)(-2) + (1)(-1) + (4)(3)}{\sqrt{(-2)^2 + (-1)^2 + (3)^2}}$$

$$= \frac{2 - 1 + 12}{\sqrt{14}} = \boxed{\frac{13}{\sqrt{14}}}$$

3. (9 pts) Consider the polar curve $r = 2\sin(\theta) + 3$. (Simplify your answers in this problem!)

(a) (3 pts) Find the (x, y) coordinates of the point that corresponds to $\theta = \pi/6$ on this curve.

$$r = 2\sin\left(\tfrac{\pi}{6}\right) + 3 = 2 \cdot \left(\tfrac{1}{2}\right) + 3 = 4$$

$$x = r\cos\theta = 4\cos(\pi/6) = 4\tfrac{\sqrt{3}}{2} = 2\sqrt{3}$$

$$y = r\sin\theta = 4\sin(\pi/6) = 4\tfrac{1}{2} = 2$$

$$\boxed{(x,y) = (2\sqrt{3}, 2)}$$

(b) (6 pts) The curve has one negative x-intercept.
Find the equation for the tangent line at the negative x-intercept.
(Put your final answer in the form $y = mx + b$.)

FACING NEGATIVE X-AXIS $\Rightarrow$ $\theta = \pi$

$$\theta = \pi \Rightarrow r = 2\sin(\pi) + 3 = 3 \Rightarrow (x,y) = (-3, 0)$$

$$\frac{dr}{d\theta} = 2\cos\theta \Big|_{\theta=\pi} = -2$$

$$\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}\Bigg|_{\theta=\pi} = \frac{(-2)\sin(\pi) + (3)\cos(\pi)}{(-2)\cos(\pi) - (3)\sin(\pi)}$$

$$= \frac{(-2)(0) + (3)(-1)}{(-2)(-1) - (3)(0)} = \frac{-3}{2} = -\frac{3}{2}$$

$$\boxed{\begin{aligned} y &= -\tfrac{3}{2}(x - -3) + 0 \\ y &= -\tfrac{3}{2}(x+3) \\ y &= -\tfrac{3}{2}x - \tfrac{9}{2} \end{aligned}}$$

4. (15 points) A curve is given by the vector function $\mathbf{r}(t) = \langle 2t, 5, t^2 - t \rangle$.

(a) (5 pts) There is one point on the curve at which the tangent line is parallel to the xy-plane. Give the equation for the tangent line at this point.

$\vec{r}'(t) = \langle 2, 0, 2t-1 \rangle$

PARALLEL TO XY-PLANE $\Rightarrow$ $2t - 1 \overset{?}{=} 0 \Rightarrow t = \frac{1}{2}$

$\vec{r}(\frac{1}{2}) = \langle 2(\frac{1}{2}), 5, (\frac{1}{2})^2 - (\frac{1}{2}) \rangle = \langle 1, 5, \frac{1}{4} - \frac{1}{2} \rangle = \langle 1, 5, -\frac{1}{4} \rangle$

$\vec{r}'(\frac{1}{2}) = \langle 2, 0, 0 \rangle$

$$\boxed{\begin{aligned} x &= 1 + 2t \\ y &= 5 \\ z &= -\tfrac{1}{4} \end{aligned}}$$

(b) (5 pts) Find the curvature of the curve at the point $(-4, 5, 6)$. (Leave in exact form).

$(x, y, z) = (-4, 5, 6) \Rightarrow -4 = 2t,\ 6 = t^2 - t \Rightarrow t = -2$

$\vec{r}'(-2) = \langle 2, 0, -5 \rangle$

$\vec{r}''(t) = \langle 0, 0, 2 \rangle$

$$\vec{r}'(-2) \times \vec{r}''(-2) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 0 & -5 \\ 0 & 0 & 2 \end{vmatrix}$$

$= (0-0)\vec{i} - (4-0)\vec{j} + (0-0)\vec{k}$

$= \langle 0, -4, 0 \rangle$

$$K(-2) = \frac{|\vec{r}'(-2) \times \vec{r}''(-2)|}{|\vec{r}'(-2)|^3} = \frac{4}{(4+25)^{3/2}} = \boxed{\frac{4}{(29)^{3/2}}}$$

(c) (5 pts) A second curve is given by the vector function $\mathbf{h}(u) = \langle 7 + u, 2u + 11, u^2 + u - 4 \rangle$. The two curves have one point of intersection. Find the angle of intersection, θ. (Round to the nearest degree).

INTERSECTION: $7 + u = 2t \longrightarrow 7 - 3 = 4 \overset{?}{=} 2t \Rightarrow t = 2$

$2u + 11 = 5 \Rightarrow u = -3$

$u^2 + u - 4 = t^2 - t$ check $\longrightarrow (-3)^2 + (-3) - 4 \overset{\checkmark}{=} 2 \overset{\checkmark}{=} (2)^2 - 2$

POINT OF INTERSECTION IS $(4, 5, 2)$ when $t = 2$ and $u = -3$

DIRECTIONS: $\vec{r}'(2) = \langle 2, 0, 2(2) - 1 \rangle = \langle 2, 0, 3 \rangle$

$\vec{h}'(u) = \langle 1, 2, 2u + 1 \rangle \Rightarrow h'(-3) = \langle 1, 2, -5 \rangle$

$$\cos\theta = \frac{\langle 2,0,3 \rangle \cdot \langle 1,2,-5 \rangle}{\sqrt{4+0+9}\,\sqrt{1+4+25}} = \frac{2-15}{\sqrt{13}\,\sqrt{30}} \Rightarrow \theta = \cos^{-1}\left(\frac{-13}{\sqrt{13}\,\sqrt{30}}\right) \approx \boxed{131^\circ}$$

also will accept $\boxed{49^\circ}$ ← acute angle