$\boxed{14}$ $\vec{r}(t) = \sin(t)\,\vec{i} + \cos(t)\,\vec{j} + t\,\vec{k}$

$\vec{r}\,'(t) = \cos(t)\,\vec{i} - \sin(t)\,\vec{j} + \vec{k}$ $\qquad \vec{r}\,'(1) = \cos(1)\,\vec{i} - \sin(1)\,\vec{j} + \vec{k}$

$\vec{r}\,''(t) = -\sin(t)\,\vec{i} - \cos(t)\,\vec{j}$ $\qquad \vec{r}\,''(1) = -\sin(1)\,\vec{i} - \cos(1)\,\vec{j}$

$$a_n = \frac{|\vec{r}\,'(1) \times \vec{r}\,''(1)|}{|\vec{r}\,'(1)|} = \frac{|\langle 0 - -\cos(1), -\sin(1) - 0, \underbrace{-\cos^2(1) - \sin^2(1)}_{=-1}\rangle|}{|\langle \cos(1), -\sin(1), 1\rangle|}$$

$$= \sqrt{\frac{\cos^2(1) + \sin^2(1) + 1}{\cos^2(1) + \sin^2(1) + 1}} = \boxed{1}$$

$\boxed{15}$ (a) $\vec{r}(t) = \langle \cos(t), \cos(t), \sqrt{2}\sin(t)\rangle$

$$\boxed{\vec{r}\,'(t) = \langle -\sin(t), -\sin(t), \sqrt{2}\cos(t)\rangle}$$

(b)

$$s(t) = \int_0^t |\vec{r}\,'(u)|\,du = \int_0^t \sqrt{\underbrace{\sin^2(u) + \sin^2(u) + 2\cos^2(u)}_{2(\sin^2(u)+\cos^2(u))=2}}\,du$$

$$\Rightarrow s(t) = \int_0^t \sqrt{2}\,du = \sqrt{2}\,u\Big|_0^t = \sqrt{2}\,t$$

$$s = \sqrt{2}\,t \Rightarrow \boxed{t = \frac{1}{\sqrt{2}}\,s = \frac{\sqrt{2}}{2}\,s}$$

$$\boxed{\vec{r}(t(s)) = \left\langle \cos\left(\tfrac{\sqrt{2}}{2}s\right), \cos\left(\tfrac{\sqrt{2}}{2}s\right), \sqrt{2}\sin\left(\tfrac{\sqrt{2}}{2}s\right)\right\rangle}$$

(c) $\left(\frac{1}{2}, \frac{1}{2}, \sqrt{\frac{3}{2}}\right) = (\cos(t), \cos(t), \sqrt{2}\sin(t)) \Rightarrow t = \pi/3$

(i) TANGENT LINE: direction $\vec{v} = \vec{r}\,'(\pi/3) = \langle -\sqrt{3}/2, -\sqrt{3}/2, \sqrt{2}/2\rangle$

$$x = 1/2 - \tfrac{\sqrt{3}}{2}t, \quad y = 1/2 - \tfrac{\sqrt{3}}{2}t, \quad z = \sqrt{\tfrac{3}{2}} + \tfrac{\sqrt{2}}{2}t$$

(ii) CURVATURE:

$$\vec{T}(t) = \frac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|} = \frac{\langle -\sin(t), -\sin(t), \sqrt{2}\cos(t)\rangle}{\sqrt{\sin^2(t) + \sin^2(t) + 2\cos^2(t)}}$$

$$= \left\langle -\tfrac{\sqrt{2}}{2}\sin(t), -\tfrac{\sqrt{2}}{2}\sin(t), \cos(t)\right\rangle$$

$$\vec{T}(s) = \left\langle -\tfrac{\sqrt{2}}{2}\sin\left(\tfrac{\sqrt{2}}{2}s\right), -\tfrac{\sqrt{2}}{2}\sin\left(\tfrac{\sqrt{2}}{2}s\right), \cos\left(\tfrac{\sqrt{2}}{2}s\right)\right\rangle$$

$$\kappa = \left|\frac{d\vec{T}}{ds}\right| = \left|\left\langle -\tfrac{1}{2}\cos\left(\tfrac{\sqrt{2}}{2}s\right), -\tfrac{1}{2}\cos\left(\tfrac{\sqrt{2}}{2}s\right), -\tfrac{\sqrt{2}}{2}\sin\left(\tfrac{\sqrt{2}}{2}s\right)\right\rangle\right|$$

$$= \sqrt{1/4\cos^2\left(\tfrac{\sqrt{2}}{2}s\right) + 1/4\cos^2\left(\tfrac{\sqrt{2}}{2}s\right) + 1/2\sin^2\left(\tfrac{\sqrt{2}}{2}s\right)}$$

$$\boxed{\kappa = \sqrt{1/2} = \sqrt{2}/2}$$

(iii) $$\vec{N}(t) = \frac{\vec{T}\,'(t)}{|\vec{T}\,'(t)|} = \frac{\langle -\frac{\sqrt{2}}{2}\cos(t), -\frac{\sqrt{2}}{2}\cos(t), -\sin(t)\rangle}{\underbrace{\sqrt{1/2\cos^2(t) + 1/2\cos^2(t) + \sin^2(t)}}_{=1}}$$

$$\vec{B}(t) = \vec{T}(t) \times \vec{N}(t) = \left\langle \tfrac{\sqrt{2}}{2}\sin^2(t) + \tfrac{\sqrt{2}}{2}\cos^2(t), -\tfrac{\sqrt{2}}{2}\cos^2(t) - \tfrac{\sqrt{2}}{2}\sin^2(t), 0\right\rangle$$

$$= \left\langle \tfrac{\sqrt{2}}{2}, -\sqrt{2}/2, 0\right\rangle$$

OSCULLATING PLANE: $\vec{n} = \vec{B}(\frac{\pi}{3}) = \langle \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, 0 \rangle$

$\langle \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, 0 \rangle \cdot \langle x-\frac{1}{2}, y-\frac{1}{2}, z-\frac{\sqrt{2}}{2} \rangle = 0$

$\frac{\sqrt{2}}{2}(x-\frac{1}{2}) - \frac{\sqrt{2}}{2}(y-\frac{1}{2}) = 0$

$x - \frac{1}{2} - (y - \frac{1}{2}) = 0$ $\boxed{x-y=0}$

(iv) NORMAL PLANE: $\vec{n} = \vec{r}'(\frac{\pi}{3}) = \langle -\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \rangle$

$\langle -\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \rangle \cdot \langle x-\frac{1}{2}, y-\frac{1}{2}, z-\frac{\sqrt{2}}{2} \rangle = 0$

$$\boxed{-\frac{\sqrt{2}}{2}(x-\frac{1}{2}) - \frac{\sqrt{2}}{2}(y-\frac{1}{2}) + \frac{\sqrt{2}}{2}(z-\frac{\sqrt{2}}{2}) = 0}$$

**16** $\vec{r}(t) = \langle 3+t, 2+\ln(t), 7+t^2 \rangle$ $\vec{r}'(t) = \langle 1, \frac{1}{t}, 2t \rangle$

Tangent Line: $\langle x, y, z \rangle = \langle 3+t, 2+\ln(t), 7+t^2 \rangle + u \langle 1, \frac{1}{t}, 2t \rangle$ can't use t here already in use

What value of $t$ will make it so the line goes through $(7,5,14)$?

(i) $7 = 3+t+u \Rightarrow 4 = t+u \Rightarrow u = 4-t$

(ii) $5 = 2+\ln(t) + \frac{u}{t}$

(iii) $14 = 7+t^2+2ut$

(i) & (iii) $\Rightarrow 14 = 7+t^2+2(4-t)t$

$7 = t^2+8t-2t^2$

$t^2-8t+7=0$ $(t-1)(t-7)=0$

$t=1$ or $t=7$

$u=4-1=3$ $u=4-7=-3$

check (ii) $5 = 2+\ln(t) + \frac{u}{t}$

$3 = \ln(t) + \frac{u}{t}$

$t=1, u=3$ work

$t=7, u=-3$ does not

$$\boxed{t=1}$$

**17** $\vec{v}(t) = \int \vec{a}(t)\,dt = \int \vec{i} - 12t^2\vec{j} + 2t\vec{k}\,dt = (t+c_1)\vec{i} + (-4t^3+c_2)\vec{j} + (t^2+c_3)\vec{k}$

$\vec{v}(0) = 2\vec{j} \Rightarrow c_1 = 0, c_2 = 2, c_3 = 0$

$\vec{v}(t) = t\vec{i} + (-4t^3+2)\vec{j} + t^2\vec{k}$

$\vec{r}(t) = \int \vec{v}(t)\,dt = (\frac{1}{2}t^2+d_1)\vec{i} + (-t^4+2t+d_2)\vec{j} + (\frac{1}{3}t^3+d_3)\vec{k}$

$\vec{r}(0) = \vec{i}+\vec{k} \Rightarrow d_1 = 1, d_2 = 0, d_3 = 1$

$$\boxed{\vec{r}(t) = (\frac{1}{2}t^2+1)\vec{i} + (-t^4+2t)\vec{j} + (\frac{1}{3}t^3+1)\vec{k}}$$

$\boxed{18}$ $f(x,y) = e^{3x+5y-1}$

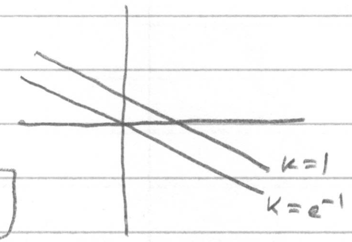
(a) $k = e^{-1} \Rightarrow e^{-1} = e^{3x+5y-1} \Rightarrow -1 = 3x+5y-1$

$0 = 3x+5y$ $\quad y = -\frac{3}{5}x$

LINES

$k = 1 \Rightarrow 1 = e^{3x+5y-1} \Rightarrow 3x+5y-1=0$ $\quad y = -\frac{3}{5}x + \frac{1}{5}$

There are no points corresponding to $k \le 0$.




(b) $\boxed{f_x(x,y) = 3e^{3x+5y-1}}$ $\boxed{f_y(x,y) = 5e^{3x+5y-1}}$

(c) $f_x(2,-1) = 3e^{6-5-1} = 3e^{-3}$

$f_y(2,-1) = 5e^{-3}$

TANGENT PLANE: $\boxed{z-1 = 3e^{-3}(x-2) + 5e^{-3}(y--1)}$

(d) $L(x,y) = 1 + 3e^{-3}(x-2) + 5e^{-3}(y+1)$

$f(1.8,-0.9) \approx 1 + 3e^{-3}(1.8-2) + 5e^{-3}(-0.9+1)$

$= 1 + 3e^{-3}(-0.2) + 5e^{-3}(0.1)$

$= 1 - 0.6e^{-3} + 0.5e^{-3} = \boxed{1 - 0.1e^{-3}}$

$\boxed{19}$ $f(x,y) = x^3 + y^2 + 2xy$

(i) $f_x(x,y) = 3x^2 + 2y \overset{?}{=} 0$

(ii) $f_y(x,y) = 2y + 2x = 0 \Rightarrow y = -x$

(i) & (ii) $3x^2 + 2(-x) = 0 \Rightarrow x(3x-2) = 0$

$x = 0$ or $3x - 2 = 0$ $\quad x = \frac{2}{3}$

$y = 0$ $\quad \boxed{(0,0) \qquad (\frac{2}{3}, -\frac{2}{3})}$ $\quad y = -\frac{2}{3}$

$f_{xx} = 6x$, $f_{yy} = 2$, $f_{xy} = 2$, $D = 12x - 4$

$(0,0) \Rightarrow D(0,0) = -4 < 0$ SADDLE POINT

$(\frac{2}{3}, -\frac{2}{3}) \Rightarrow D(\frac{2}{3}, -\frac{2}{3}) = 8 - 4 = 4 > 0$ LOCAL MIN

$f_{xx}(\frac{2}{3}, -\frac{2}{3}) = 4 > 0$

**20** $x+y=2 \Rightarrow y=-x+2$

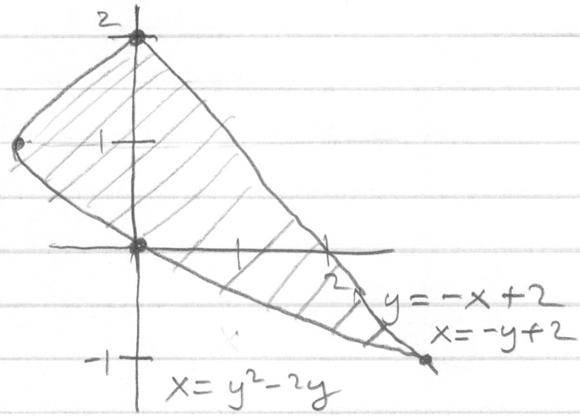
$y^2-2y-x=0 \Rightarrow y^2-2y=x$

$y(y-2)=x$

intersect $y^2-2y=2-y$

$y^2-y-2=0$

$(y-2)(y+1)=0$




$\iint_D x+y \, dA$

$-1 \le y \le 2$

$y^2-2y \le x \le -y+2$

$\int_{-1}^{2}\int_{y^2-2y}^{-y+2} x+y \, dx\, dy$

$= \int_{-1}^{2} \frac{1}{2}x^2+yx \Big|_{y^2-2y}^{-y+2} dy = \int_{-1}^{2}\Big[\Big(\frac{1}{2}(-y+2)^2+y(-y+2)\Big) - \Big(\frac{1}{2}(y^2-2y)^2+y(y^2-2y)\Big)\Big] dy$

$= \int_{-1}^{2} \frac{1}{2}(y^2-4y+4)-y^2+2y-\frac{1}{2}(y^4-4y^3+4y^2)-y^3+2y^2 \, dy$

$= \int_{-1}^{2} \frac{1}{2}y^2-\cancel{2y}+2-y^2+\cancel{2y}-\frac{1}{2}y^4+2y^3-\cancel{2y^2}-y^3+\cancel{2y^2} \, dy$

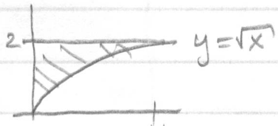
$= \int_{-1}^{2} -\frac{1}{2}y^4+y^3-\frac{1}{2}y^2+2 \, dy = -\frac{1}{10}y^5+\frac{1}{4}y^4-\frac{1}{6}y^3+2y \Big|_{-1}^{2}$

$= \Big(-\frac{1}{10}2^5+\frac{1}{4}2^4-\frac{1}{6}2^3+2(2)\Big)-\Big(-\frac{1}{10}(-1)^5+\frac{1}{4}(-1)^4-\frac{1}{6}(-1)^3+2(-1)\Big)$

$= \boxed{\frac{99}{20} \doteq 4.95}$

**21** (a) $0 \le x \le 4$

$\sqrt{x} \le y \le 2$




$0 \le y \le 2$

$0 \le x \le y^2$

$\int_0^2\int_0^{y^2} xy \, dx\, dy$

(b) $\int_0^2\int_0^{y^2} xy \, dx\, dy = \int_0^2 \frac{1}{2}x^2y \Big|_0^{y^2} dy = \frac{1}{2}\int_0^2 y^5 \, dy$

$= \frac{1}{12}y^6 \Big|_0^2 = \frac{2^6}{12} = \boxed{\frac{16}{3}}$

$\int_0^4\int_{\sqrt{x}}^{2} xy \, dy\, dx = \int_0^4 \frac{1}{2}xy^2 \Big|_{\sqrt{x}}^{2} dx = \int_0^4 2x-\frac{1}{2}x^2 \, dx$

$= x^2-\frac{1}{6}x^3\Big|_0^4 = 4^2-\frac{1}{6}4^3 = 16\left(1-\frac{4}{6}\right) = \boxed{\frac{16}{3}}$

$\boxed{22}$ $\int_0^3 \int_0^{9-x^2} \frac{x e^{3y}}{9-y} \, dy \, dx$ CAN'T INTEGRATE, NEED TO REVERSE ORDER

$0 \le x \le 3$

$0 \le y \le 9 - x^2$

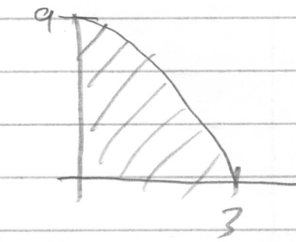
$y = 9 - x^2$

$x^2 = 9 - y$

$x = \pm\sqrt{9-y}$

$0 \le y \le 9$

$0 \le x \le \sqrt{9-y}$

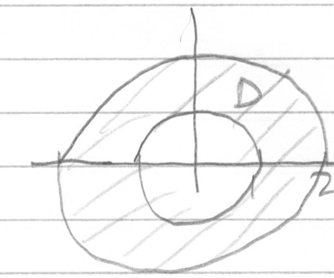



$\int_0^9 \int_0^{\sqrt{9-y}} \frac{e^{3y}}{9-y} \, x \, dx \, dy$

$\int_0^9 \frac{e^{3y}}{9-y} \frac{1}{2} x^2 \Big|_0^{\sqrt{9-y}} dy = \int_0^9 \frac{e^{3y}}{\cancel{9-y}} \frac{1}{2} \cancel{(9-y)} \, dy$

$= \frac{1}{6} e^{3y} \Big|_0^9 = \boxed{\frac{1}{6} e^{27} - \frac{1}{6}} = \frac{1}{6}\left(e^{27} - 1\right)$

$\boxed{23}$ $\iint_D y^2 \, dA$




$0 \le \theta \le 2\pi$

$1 \le r \le 2$

$\int_0^{2\pi} \int_1^2 r^2 \sin^2(\theta) \, r \, dr \, d\theta$

$\int_0^{2\pi} \sin^2(\theta) \; \frac{1}{4} r^4 \Big|_1^2 d\theta$

$\int_0^{2\pi} \sin^2(\theta) \left( \frac{1}{4} 2^4 - \frac{1}{4} \right) d\theta$

$\frac{15}{4} \int_0^{2\pi} \sin^2(\theta) \, d\theta$

$\frac{15}{4} \int_0^{2\pi} \frac{1}{2}\left(1 - \cos(2\theta)\right) d\theta$

$\frac{15}{8} \left[ \theta - \frac{1}{2} \sin(2\theta) \Big|_0^{2\pi} \right] = \frac{15}{8}(2\pi) = \boxed{\frac{15\pi}{4}}$