Notes on the homework concerning parametric equations

Many small and simple facts come at you all at once in sections 10.1, 10.2, 13.1, and 13.2. If you took Math 124 here, then 10.1 and 10.2 should be review. But if you are new to this material, or if you have forgotten, then that will make your task of understanding this material that much more difficult. So I decided to put together a little guide to help you get through the homework in 10.1/13.1 and 10.2/13.2 and to emphasize the fundamental concepts in each problem. It would also be wise to visit the math study center and to make good use of quiz section.

Homework notes on 10.1/13.1:

1. Problems 1-3: You should know how to eliminate the parameter. And you should know how to plot a few points to get a rough idea about the graph. Remember our two methods for eliminating the parameter
   (a) If you see $\sin(BLAH)$ and $\cos(BLAH)$ separately in x and y, try squaring and adding together. Here is a quick example, $x = 3\sin(4t)$, $y = 2\cos(4t)$, then we have $\sin(4t) = x/3$ and $\cos(4t) = y/2$, and using the identity, $(x/3)^2 + (y/2)^2 = (\sin(4t))^2 + (\cos(4t))^2 = 1$. Thus, this motion is along the path $x^2/9 + y^2/4 = 1$, which is an ellipse.
   (b) Otherwise, try to solve for t in one equation and substitute into the other. For example, $x = 2t$, $y = t^2 + t$, then $t = x/2$, so $y = (x/2)^2 + (x/2)$.
   (c) Sometimes it is not possible to eliminate the parameter, so you just have to analyze x and y separately and plot points. Meaning make a table of (x, y) points to plot by picking variuos values of t.
2. Problems 4-5: Use a process of elimination and plot points. Look for things like
   (a) Is x or y ever zero? That would tell if the xy-plane graph should have any y or x intercepts. Are they zero at the same value of t? That would mean the xy-graph has to go through the origin.
   (b) What is the range of values x can take? If you know that $x > 1$, then that means in the xy-plane the curve has to always be to the right of $x = 1$. Similarly, what is the range of values y can take?
   (c) Plot several points! You can make a little table for $t = 0$, $t = 1$, $t = 2$, ... where you compute x and y (by plugging in or looking at the information given). Then plot these points. That should make it easy for you to see which graphs can't be correct. Typically you would only have to plot one or two points and you would immediately see the answer.
3. Problems 6: Use trigonometry to find relationships between the lengths of the various line segments in the problem. These are giving you situations where parameterization naturally arises, and in these situations you can use trig to solve for the precise parameterization. (Hint: The triangle OAB has a right angle at A).
4. Problems 7-11: Some basic function type questions in the setting of vector functions.
   (a) The domain of the function is the set of allowable inputs so find the allowable inputs for the 1st component, then the 2nd, then the 3rd (and combine them into one interval of values for t that would be valid for all three).
   (b) Do the limit componentwise (meaning do three limits and the resulting vector is your answer).
   (c) For the other questions plot points.

5. Problems 12-13: You are visualizing 3D curves. Eliminate the parameter like we did in class to find the surface of motion. That should quickly tell you which picture is correct.
6. Problems 14-15: Working with parametric curves and questions about intersections. Combine the conditions on x, y and z, then solve for t (and use t to go back and get the (x, y, z) coordinates).
7. Problems 16-17: Parameterizing a curve. There are infinitely many answers to these questions! You need to find some parameterization that gives you the two equations when you eliminate the parameter. (Typically, one of the fastest ways of finding such a parameterization is to let $x = t$, then solve for the resulting equations for y and z. Or you could let $y = t$ and work from there. Or you could let $z = t$ and work from there. But there are many other options).
8. Problem 18-19: You MUST use different variables for the parameters for two curves if you are trying to find if the paths of the curves every 'intersect'. If you use the same parameter, then you are finding if the particles in motion every 'collide' (in which case you use the same parameter). We discuss the same issue when we were talking about lines intersecting.

Homework notes on 10.2/13.2:

1. Problems 1-5: Most of this should be review from Math 124. You are using the fact that $dy/dx = (dy/dt)/(dx/dt)$ which is the slope of the tangent line (in terms of t). The only thing that is new in these five problems is that you have to compute the second derivative once, which is $d/dx(dy/dx) = (d/dt(dy/dx))/(dx/dt)$. We did an example of this in class and there are examples in the book.
2. Problem 6: This is also from Math 124. The given point may or may not be on the curve. The question is asking you to find all tangent lines to the curve such that the tangent line also passes through the given point (in other words, the point (a, b) at which the line is tangent is unknown, but you know the line goes through this other given point). So you are looking for all values of t such that: The line from $(x(t), y(t))$ to the given point $(12, 8)$ (your numbers may be different) has the same slope as the tangent line at t. You should know the expression for the slope of a line between two points and you should know the expression for the equation of the tangent line at t. Now set them equal and solve for t. This type of problem is something we did a lot of in our Math 124 classes.
3. Problems 7-12: These introduce you (in 2-dimensions) to the idea of position and tangent vectors.
4. Problems 13: This has you compute a tangent vector and use it to find a tangent line. The tangent line is a fundamental object, you want a problem like this to be routine.
5. Problem 14-15: You want to find the angles between the tangent vectors at the point of intersection.
6. Problems 16-17: Integrate each component separately (we'll talk more about when we would use this in section 13.4).
7. Problem 18-19: This is showing you that the product rule works for dot products and cross-products. The problem is exposing you to the rules. I just want you to find the derivative in the fastest way you can. The answer for 18 will be about three terms so moderate in length, but the answer to 19 will be large (this is a bit tedious, so give yourself plenty of paper and be organized on this problem).
8. Problems 20-22: Computing arc lengths. For the last one, please, please, plesae draw a picture and label points. The curve is being traversed more than once!