Math 126 Exam 2 Quick Review

13.3: Measurement on 3D Curves

1. Arc Len. $= \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}\, dt$.
2. $\kappa(t) = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$ (in 2D, $\kappa(x) = \frac{|f''(x)|}{(1+(f'(x))^2)^{3/2}}$)
3. $\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$, $\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|}$, $\mathbf{B} = \mathbf{T} \times \mathbf{N}$.
   Conceptual reminder: $\mathbf{r}'(t_0)$, $\mathbf{r}''(t_0)$, $\mathbf{T}(t_0)$, and $\mathbf{N}(t_0)$ are all on the same plane (the osculating plane).
4. Tangent Line:
   Through $\mathbf{r}(t_0)$ in direction of $\mathbf{r}'(t_0)$.
5. Normal Plane:
   Through (x_0, y_0, z_0) with normal in direction of $\mathbf{r}'(t_0)$.
6. Osculating Plane:
   Through (x_0, y_0, z_0) with normal in direction of $\mathbf{B}(t_0)$.

13.4: Velocity and Acceleration

1. If t is time, then
   $\mathbf{r}(t) =$ position
   $\mathbf{v}(t) = \mathbf{r}'(t)$ is velocity, $|\mathbf{v}(t)|$ is speed, and
   $\mathbf{a}(t) = \mathbf{r}''(t)$ is acceleration.
   Be able to go from position to acceleration and acceleration to position. (Be careful with constants of integration).
2. $a_T = \frac{\mathbf{r}'(t) \cdot \mathbf{r}''(t)}{|\mathbf{r}'(t)|}$, $a_N = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|}$

14.1, 14.3, 14.4: Partials

1. Sketch a domain and sketch level curves.
2. Compute partial derivatives and understand what they represent.
   $f_x(x_0, y_0) =$ 'slope in x-direction'
   $f_y(x_0, y_0) =$ 'slope in y-direction'
3. Find a tangent plane, a linearization, and the total differential:

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0).$$

$$L = z_0 + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0).$$

$$dz = f_x(x_0, y_0)dx + f_y(x_0, y_0)dy$$

14.7: Critical points and max/min

1. Find critical points:
   Set $f_x(x, y) = 0$ and $f_y(x, y) = 0$,
   then COMBINE and solve (check your answers).
2. Classify critical points (second derivative test).
   $D = f_{xx}f_{yy} - f_{xy}^2$.
   If $D > 0$ and $f_{xx} > 0$, then local min.
   If $D > 0$ and $f_{xx} < 0$, then local max.
   If $D < 0$, then saddle point.
3. Find the absolute max/min over a region.
   (a) Find critical points.
   (b) Find the critical numbers on each boundary.
   (c) Evaluate the original function at all critical points inside and critical numbers on the boundaries and all corners.

15.1-15.4: Double Integrals.

1. $\displaystyle\iint_D f(x,y)\,dA =$
   signed volume 'above' the region D in the xy-plane and 'below' $f(x,y)$.

2. Other applications:
   $\displaystyle\iint_D 1\,dA = \text{area of } D.$

   $\displaystyle\frac{1}{\text{Area of D}}\iint_D f(x,y)\,dA =$
   average value of $f(x,y)$.

3. To set up a double integral from a description:
   (a) Solving for integrand(s) ($z =???$).
   (b) Draw given xy-equations in xy-plane.
   (c) Draw any xy-equations that occur from intersections of the surfaces (*i.e.* $z = f(x,y)$ with $z = 0$ or the intersection of two given surfaces).
   (d) Set up the double integral(s) using the region for D.

4. Options for set up:
   $$\iint_D f(x,y)\,dA = \int_a^b \int_{g(x)}^{h(x)} f(x,y)\,dydx$$
   $$= \int_c^d \int_{p(y)}^{q(y)} f(x,y)\,dxdy$$
   $$= \int_\alpha^\beta \int_{w(\theta)}^{v(\theta)} f(r\cos(\theta), r\sin(\theta))\,rdrd\theta$$

5. Be able to DRAW the region D if given a double integral that is already set up and then reverse the order.