Local Max/Min for One Variable Function

A **critical value** is any number \( x = c \) such that \( f'(c) = 0 \) or \( f'(c) \) does not exist.

The Second Derivative Test:
If \( x = c \) is a critical value, then

1. \( f''(c) > 0 \Rightarrow x = c \) gives a local min.
2. \( f''(c) < 0 \Rightarrow x = c \) gives a local max.
3. \( f''(c) = 0 \Rightarrow \) inconclusive (other methods needed).
Local Max/Min for Two Variable Function

A **critical point** is any point \((x, y) = (a, b)\) such that \(f_x(a, b) = 0\) AND \(f_y(a, b) = 0\) (both) or \(f_x(a, b)\) DNE or \(f_y(a, b)\) DNE.

**The Second Derivative Test:**
If \((a, b)\) is a critical point, then define

\[
D = D(a, b) = f_{xx}(a, b)f_{yy}(a, b) - [f_{xy}(a, b)]^2.
\]

1. \(D > 0, f_{xx} > 0 \Rightarrow (a, b)\) gives a local min.
2. \(D > 0, f_{xx} < 0 \Rightarrow (a, b)\) gives a local max.
3. \(D < 0 \Rightarrow (a, b)\) gives a saddle point.
4. \(D = 0 \Rightarrow \) inconclusive (use a contour map).
Global Max/Min for One Variable Function

To find abs. max/min of $f(x)$ on a closed interval:

1. Find critical numbers.
2. Evaluate $f(x)$ at the critical numbers.
3. Evaluate $f(x)$ at the endpoints.
   
   Biggest output = absolute max.
   Smallest output = absolute min.
Global Max/Min for Two Variable Function

To find abs. max/min of $f(x, y)$ on a closed region:

1. Find critical points.

2. Over each boundary curve:
   (a) Find $xy$ equation.
   (b) Substitute boundary equation into $f(x, y)$ to get a one variable function for $z$.
   (c) Use Calculus 1 methods to find critical numbers and endpoints on that boundary.

3. Evaluate $f(x, y)$ at the critical points inside the region.

4. Evaluate $f(x, y)$ at the critical numbers and endpoints on each boundary.
   Biggest output = absolute max.
   Smallest output = absolute min.
Applied Optimization for single or multi-variable functions

In applied problems, we have to set up the function to optimize. Here are things I always suggest:

1. VISUALIZE/LABEL: Draw a good picture and label everything with variables.

2. WHAT IS GIVEN?: Write down all the given constraints.

3. WHAT TO OPTIMIZE?: Write down a formula for that quantity. Then, using the given facts, find a function for the quantity that you want to optimize.

4. DOMAIN? Over what interval does the problem make sense

5. USE CALCULUS: Find the methods just discussed.

6. JUSTIFY/VERIFY: Make sure you actually did find the a max or min as desired.