Integrating Powers of Trig

\[ \int \cos(ax) \, dx = \frac{1}{a} \sin(ax) + C \]
\[ \int \sin(ax) \, dx = -\frac{1}{a} \cos(ax) + C \]

Even powers (half-angle identity):

\[ \int \cos^2(x) \, dx = \frac{1}{2} \int 1 + \cos(2x) \, dx = \frac{1}{2} x + \frac{1}{4} \sin(2x) + C \]
\[ \int \sin^2(x) \, dx = \frac{1}{2} \int 1 - \cos(2x) \, dx = \frac{1}{2} x - \frac{1}{4} \sin(2x) + C \]

\[ \int \cos^4(x) \, dx = \int \left[ \frac{1}{2}(1 + \cos(2x)) \right]^2 \, dx = \frac{1}{4} \int 1 + 2 \cos(2x) + \cos^2(2x) \, dx \]
\[ = \frac{1}{4} x + \frac{1}{4} \sin(2x) + \frac{1}{4} \int \cos^2(2x) \, dx \]
\[ = \frac{1}{4} x + \frac{1}{4} \sin(2x) + \frac{1}{8} \int 1 + \cos(4x) \, dx \]
\[ = \frac{1}{4} x + \frac{1}{4} \sin(2x) + \frac{1}{8} x + \frac{1}{32} \sin(4x) + C \]
\[ = \frac{3}{8} x + \frac{1}{4} \sin(2x) + \frac{1}{32} \sin(4x) + C \]

You can do \( \sin^4(x) \) and \( \sin^2(x) \cos^2(x) \) in a similar way as above.

Odd powers (identity then substitution):

\[ \int \cos^3(x) \, dx = \int \cos^2(x) \cos(x) \, dx = \int (1 - \sin^2(x)) \cos(x) \, dx \]
then use \( u = \sin(x) \) to get
\[ \int 1 - u^2 \, du = u - \frac{1}{3} u^3 + C \]
so
\[ \int \cos^3(x) \, dx = \sin(x) - \frac{1}{3} \sin^3(x) + C \]

Here is another example

\[ \int \cos^2(x) \sin^3(x) \, dx = \int \cos^2(x) \sin^2(x) \sin(x) \, dx = \int \cos^2(x)(1 - \cos^2(x)) \sin(x) \, dx \]
then use \( u = \cos(x) \) to get
\[ \int -u^2(1 - u^2) \, du = -\frac{1}{3} u^3 + \frac{1}{5} u^5 + C \]
so
\[ \int \cos^2(x) \sin^3(x) \, dx = -\frac{1}{3} \cos^3(x) + \frac{1}{5} \cos^5(x) + C \]