1. (8 points) Let \( \mathbf{r}(t) = (2t - 1)\mathbf{i} + t^2 \mathbf{j} + 2\sqrt{t} \mathbf{k} \). Find all times \( t \) when the tangential component of acceleration is zero.

The tangential component of acceleration at time \( t \) is \( a_T = \frac{\mathbf{r}'(t) \cdot \mathbf{r}''(t)}{||\mathbf{r}'(t)||} \).

Thus \( a_T = 0 \) when \( \mathbf{r}'(t) \cdot \mathbf{r}''(t) = 0 \).

\[
\mathbf{r}'(t) \cdot \mathbf{r}''(t) = \langle 2, 2t, t^{-1/2} \rangle \cdot \langle 0, 2, -\frac{1}{2}t^{-3/2} \rangle = 4t - \frac{1}{2}t^{-2}
\]

Solving \( 4t - \frac{1}{2}t^{-2} = 0 \) gives \( 8t^3 - 1 = 0 \) so \( t = \frac{1}{2} \) is the only solution.

2. (6 points) Find the equation of the tangent plane of the function \( F(x, y) = \frac{3y - 2}{5x + 7} \) at the point \((-1, 1)\).

\[
F_x(x, y) = -5 \cdot \frac{3y - 2}{(5x + 7)^2} \quad \text{so} \quad F_x(-1, 1) = -\frac{5}{4}
\]

\[
F_y(x, y) = \frac{3}{5x + 7} \quad \text{so} \quad F_y(-1, 1) = \frac{3}{2}
\]

\[
F(-1, 1) = \frac{1}{2}
\]

The equation of the tangent plane is \( z - \frac{1}{2} = -\frac{5}{4}(x + 1) + \frac{3}{2}(y - 1) \).
(14 points) Evaluate the following double integrals.

(a) (7 points) \( \iint_R xy \sin(x^2y) \,dA \), \( R = [0, 1] \times [0, \pi/2] \)

Use Fubini’s theorem to convert this to an iterated integral.

\[
\iint_R xy \sin(x^2y) \,dA = \int_0^{\pi/2} \int_0^1 xy \sin(x^2y) \,dx \,dy
\]
\[
= \int_0^{\pi/2} \left( \left. -\frac{1}{2} \cos(x^2y) \right|_0^1 \right) \,dy
\]
\[
= \int_0^{\pi/2} \left( \frac{1}{2} - \frac{1}{2} \cos(y) \right) \,dy
\]
\[
= \left. \frac{1}{2}y - \frac{1}{2} \sin(y) \right|_0^{\pi/2}
\]
\[
= \frac{\pi}{4} - \frac{1}{2}
\]

(b) (7 points) \( \iint_D y^2 e^{xy} \,dA \), \( D = \{ (x, y) \mid 0 \leq y \leq 3, \ 0 \leq x \leq y \} \)

\[
\iint_D y^2 e^{xy} \,dA = \int_0^3 \int_0^y y^2 e^{xy} \,dx \,dy
\]
\[
= \int_0^3 \left( e^{xy} \right|_0^y \,dy
\]
\[
= \int_0^3 ye^{y^2} - y \,dy
\]
\[
= \left. \frac{1}{2}e^{y^2} - \frac{1}{2}y^2 \right|_0^3
\]
\[
= \frac{1}{2}e^9 - 5
\]
(12 points) You wish to build a rectangular box with no top with volume 6 ft³. The material for the bottom is metal and costs $3.00 a square foot. The sides are wooden and cost $2.00 a square foot. Calculate the dimensions of the box with minimum cost. Use the Second Derivative test to verify that your answer is indeed a minimum.

Label the sides as shown. The cost is \( C = 3xy + 4xz + 4yz \). The volume constraint is \( xyz = 6 \).

From the constraint we get \( z = \frac{6}{xy} \). Substituting into the cost function gives the objective function \( C(x, y) = 3xy + \frac{24}{y} + \frac{24}{x} \).

Now calculate the partial derivatives and set them equal to zero.

\[
C_x(x, y) = 3y - \frac{24}{x^2} = 0 \quad \text{gives} \quad y = \frac{8}{x^2}.
\]

\[
C_y(x, y) = 3x - \frac{24}{y^2} = 0 \quad \text{gives} \quad x = \frac{8}{y^2}.
\]

Combining, we get \( x = \frac{8}{(8/x^2)^2} \) or \( 8x = x^4 \). The solutions are \( x = 0, 2 \) but \( x = 0 \) makes no sense and can be discarded. If \( x = 2 \) and \( y = \frac{8}{x^2} \) we get \( y = 2 \) as well. Since \( xyz = 6 \) it follows that \( z = \frac{3}{2} \).

The dimensions of the box are \( 2 \times 2 \times \frac{3}{2} \).

To verify that this is gives the minimum cost, we must compute the second derivatives.

\[
C_{xx}(2, 2) = \frac{48}{x^3}_{x=2} = 6
\]

\[
C_{yy}(2, 2) = \frac{48}{y^3}_{y=2} = 6
\]

\[
C_{xy}(x, y) = 3
\]

Thus the Hessian determinant is \( 6 \cdot 6 - 3 \cdot 3 > 0 \). Since \( C_{xx}(2, 2) > 0 \), the point \( (2, 2) \) gives a minimum of the cost function by the Second Derivative Test.
A table of values is given for a function $g(x, y)$ defined on $R = [0, 1] \times [1, 4]$. (For example, $g(1, 4) = 9.4$.) Use the table to find a linear approximation to $g(x, y)$ near $(0.5, 3)$. Use it to approximate $g(0.6, 2.8)$. Carefully explain all your reasoning.

We need to approximate the partial derivatives $g_x(0.5, 3)$ and $g_y(0.5, 3)$. There are several correct ways to do this. I will choose one.

$I$ approximate $g_x(0.5, 3)$ with the slope of the secant line from $(0.5, 3, 5.5)$ to $(0.75, 3, 5.8)$.

The slope is

$$\Delta z \over \Delta x = \frac{5.8 - 5.5}{0.75 - 0.5} = 1.2.$$ 

$I$ approximate $g_y(0.5, 3)$ with the slope of the secant line from $(0.5, 3, 5.5)$ to $(0.5, 2.5, 4.2)$.

The slope is

$$\Delta z \over \Delta y = \frac{4.2 - 5.5}{2.5 - 3} = 2.6.$$ 

The linear approximation $L(x, y) = 5.5 + 1.2(x - 0.5) + 2.6(y - 3)$.

$L(0.6, 2.8) = 5.5 + 1.2 \cdot 0.1 - 2.6 \cdot 0.2 = 5.1$