Math 126
Exam 1
January 30, 2024

Name ______________________________________

Student ID #________________

Section ________________

HONOR STATEMENT
"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam."

SIGNATURE:_________________________________

- This exam consists of this cover, four pages of questions, and a blank "scratch sheet". If you put work on the scratch sheet and you want it to be graded, then you must clearly tell us in the problem to "see scratch page".
- You will have 50 minutes.
- You are allowed to use a Ti-30x IIS Calculator model ONLY (**no other calculators allowed**) and one 8.5 by 11 inch sheet of handwritten notes (front and back). All other sources are forbidden.
- Turn your cell phone OFF and put it away for the duration of the exam. You may not listen to headphones or earbuds during the exam.
- **You must show your work.** The correct answer with no supporting work may result in no credit.
- Leave your answer in exact form. Simplify standard trig, inverse trig, natural logarithm, and root values. Here are several examples: you should write $\sqrt{4} = 2$ and $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ and $\ln(1) = 0$ and $\tan^{-1}(1) = \frac{\pi}{4}$.
- Unless otherwise indicated, when rounding is necessary, you may round your final answer to two digits after the decimal.
- **Do not write within 1 centimeter of the edge!** Your exam will be scanned for grading.
- There may be multiple versions, you have signed an honor statement, and cheating is a hassle for everyone involved. If we find that you give an answer that is only appropriate for the other version of the exam and there is no work to support your answer, then you will get a zero on the entire exam and your work will be submitted to the academic misconduct board. JUST DO NOT CHEAT.

GOOD LUCK!

1. (12 pts)

(a) Find and simplify an equation for the surface consisting of all points (x, y, z) that are equidistant from the point $(0, 0, 2)$ and the xy-plane. Then give the precise name of this surface.

Equation: ______________________________

Name: ______________________________

(b) Consider the three points $A(1, 2, 0)$, $B(3, 0, 2)$, $C(4, 1, 0)$.

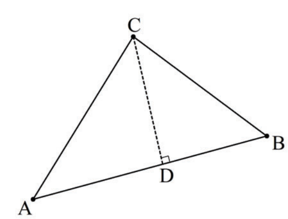
i. Find the area of the triangle formed by these three points.

Area = ____________________

ii. Find the equation of the plane containing these three points (Simplify your answer into the form $Ax + By + Cz = D$).

Plane Equation: ______________________________

iii. The point D is on the line segment from A to B and the line from D to C is perpendicular to that line segment (as shown). Find the distance from A to D and find the coordinates of the point D.



Dist from A to D: ____________________

Coordinates D: $(x, y, z) =$ ______________________________

2. (14 pts)

(a) Determine whether each statement is always true or false in $\mathbb{R}^3$.
(Put "$\times$" in the circle next to your choice)

i. ◯ TRUE ◯ FALSE : Two lines perpendicular to a given plane are parallel.

ii. ◯ TRUE ◯ FALSE : Two planes parallel to a given line are parallel.

(b) Find the equation for the plane containing the intersecting lines
$L_1 : x = 2 + 3t, y = 1 - t, z = 5 + 2t$ and $L_2 : x = 5 - u, y = 2u, z = 7 - 3u$.
(Simplify your answer into the form $Ax + By + Cz = D$).

Plane Equation: ______________________________

(c) Find parametric equations for the line of intersection of the two planes $2x - y + 3z = 10$ and $x - y + z = 2$.

Line Equations: ______________________________

3. (12 pts) Consider curve given by $\mathbf{r}_1(t) = \langle 2t+1, -3t, t^3 - t^2 + 4 \rangle$.

(a) Find equations for the tangent line to the curve at the point $(5, -6, 8)$. And give the point of intersection of this tangent line with the plane $x + y + z = 42$.

Intersection point $(x, y, z) =$ ____________________

(b) Find the angle of intersection (to the nearest degree) of the given curve $\mathbf{r}_1(t)$ and the second curve $\mathbf{r}_2(u) = \langle 4+u, -6, 4\sqrt{u^2+3} \rangle$.

Angle = ____________________ degrees

4. (12 pts)

(a) Find $\mathbf{r}(t)$ if $\mathbf{r}'(t) = 2e^{2t}\mathbf{i} + \sec^2(t)\mathbf{j} + t\sin(t^2)\mathbf{k}$ and $\mathbf{r}(0) = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$.

$\mathbf{r}(t) =$ ______________________________

(b) Find the value of x at which $f(x) = e^x$ has maximum curvature.

$x =$ ______________________________

You may use this page for scratch-work or extra room.
All work on this page will be ignored unless you write and circle "see scratch page" on the problem page and you label your work below.