1. (10 pts)

$\boxed{6}$ (a) Find the linear approximation to $f(x,y) = x^2\cos(3y) + ye^{2y} + \sqrt{x^3 + \underset{4}{\cancel{2}}y}$ at $(1,0)$.

$$f_x = 2x\cos(3y) + \frac{3x^2}{2\sqrt{x^3+4y}}$$

$$f_y = -3x^2\sin(3y) + e^{2y} + 2ye^{2y} + \frac{\overset{2}{\cancel{4}}}{\cancel{2}\sqrt{x^3+4y}}$$

$$f_x(1,0) = 2\cdot 1\cdot\cos(0) + \frac{3\cdot 1^2}{2\sqrt{1^3+4\cdot 0}} = 2 + \frac{3}{2} = \frac{7}{2} = 3.5$$

$$f_y(1,0) = 0 + e^0 + 2(0)e^0 + \frac{2}{\sqrt{1^3+0}} = 1 + 2 = 3$$

$$f(1,0) = (1)^2\cos(3\cdot 0) + 0e^0 + \sqrt{1^3+0} = 1+1 = 2$$

$$z - 2 = \frac{7}{2}(x-1) + 3y$$

$L(x,y) =$ $2 + \frac{7}{2}(x-1) + 3y$

$\boxed{4}$ (b) Use implicit differentiation to find $\frac{\partial z}{\partial x}$ for the curve defined by

$$3z + 1 = x^2ye^z + \ln(y)$$

$$3\frac{\partial z}{\partial x} = 2xye^z + x^2ye^z\frac{\partial z}{\partial x}$$

$$(3 - x^2ye^z)\frac{\partial z}{\partial x} = 2xye^z$$

$$\boxed{\frac{\partial z}{\partial x} = \frac{2xye^z}{3 - x^2ye^z}}$$

$\frac{\partial z}{\partial x} =$ $\frac{2xye^z}{3 - x^2ye^z}$

2. (14 pts)

7

(a) Find and classify all the critical points for $f(x, y) = \frac{1}{2}x^2y - x^2 - \frac{1}{2}y^2 + y$. (*Clearly give your second derivatives and labeling your critical points as max, min, or saddle points*).

+2

① $f_x = xy - 2x = 0 \to x(y-2) = 0$

② $f_y = \frac{1}{2}x^2 - y + 1 = 0$

+2

① $\nearrow x = 0 \xrightarrow{②} -y+1=0 \to y=1 \quad (0,1)$

$\searrow y = 2 \xrightarrow{②} \frac{1}{2}x^2 - 2 + 1 = 0 \to \frac{1}{2}x^2 = 1 \to x^2 = 2 \begin{cases} x = -\sqrt{2} & (-\sqrt{2}, 2) \\ x = \sqrt{2} & (\sqrt{2}, 2) \end{cases}$

+3

$f_{xx} = y - 2, \; f_{yy} = -1, \; f_{xy} = x$

$(0,1) \Rightarrow D = (-1)(-1) - (0)^2 = 1 > 0$ CONCAVE DOWN IN ALL DIRECTIONS

$(-\sqrt{2}, 2) \Rightarrow D = (0)(-1) - (-\sqrt{2})^2 = -2 < 0$

$(\sqrt{2}, 2) \Rightarrow D = (0)(-1) - (\sqrt{2})^2 = -2 < 0$

$\}$ SADDLE POINTS

List and Label: $(x, y) =$ LOCAL MAX $(0,1)$ SADDLE POINTS $(-\sqrt{2}, 2)$ $(\sqrt{2}, \sqrt{2})$

7

(b) Find the points on the hyperboloid of two sheets $3x^2 + y^2 - z^2 = -1$ that are closest to the point $(8, 2, 0)$. (*Find a two variable function for the distance, then find the critical point, no justification needed*)

+1

$\text{DIST} = \sqrt{(x-8)^2 + (y-2)^2 + z^2}$ $\quad z^2 = 1 + 3x^2 + y^2$

+2

$\Rightarrow \text{DIST} = f(x,y) = \sqrt{(x-8)^2 + (y-2)^2 + 1 + 3x^2 + y^2} \leftarrow$ MESS

+2

$f_x = \frac{1}{2\sqrt{\text{MESS}}} \cdot (2(x-8) + 6x) \stackrel{?}{=} 0 \Rightarrow 2x - 16 + 6x = 0 \Rightarrow x = 2$

$f_y = \frac{1}{2\sqrt{\text{MESS}}} \cdot (2(y-2) + 2y) \stackrel{?}{=} 0 \Rightarrow 2y - 4 + 2y = 0 \Rightarrow y = 1$

+2

$z^2 = 1 + 3(2)^2 + (1)^2$

$z^2 = 1 + 12 + 1$

List both: $(x, y, z) =$ $(2, 1, -\sqrt{14})$ $\quad (2, 1, \sqrt{14})$

3. ([redacted] pts)

$\boxed{6}$ (a) Evaluate $\iint_R y\sin(xy)\,dA$ over the rectangle $R = \{(x,y)\,|\,0 \le x \le \frac{\pi}{2}, 0 \le y \le 1\}$.

(+2) $\int_0^1 \int_0^{\pi/2} y\sin(xy)\,dx\,dy$

(+2) $= \int_0^1 y \cdot \frac{-1}{y}\cos(xy)\Big|_0^{\pi/2} dy$

$= -\int_0^1 \cos\left(\frac{\pi}{2}y\right) - 1\,dy$

(+2) $= -\left(\frac{2}{\pi}\sin\left(\frac{\pi}{2}y\right) - y\,\Big|_0^1\right)$

$= -\left(\frac{2}{\pi} - 1\right)$

OR $\int_0^{\pi/2}\int_0^1 y\sin(xy)\,dy\,dx$ $\quad u = y \quad dv = \sin(xy)\,dy$ $\quad du = dy \quad v = -\frac{1}{x}\cos(xy)$

$= \int_0^{\pi/2}\left(-\frac{y}{x}\cos(xy)\Big|_0^1 - \int_0^1 \frac{-1}{x}\cos(xy)\,dy\right)dx$

$= \int_0^{\pi/2}\left(-\frac{\cos(x)}{x} + \frac{1}{x^2}\sin(xy)\Big|_0^1\right)dx$

$= \int_0^{\pi/2}\left(\frac{-\cos(x)}{x} + \frac{\sin(x)}{x^2}\right)dx$

YUCK!

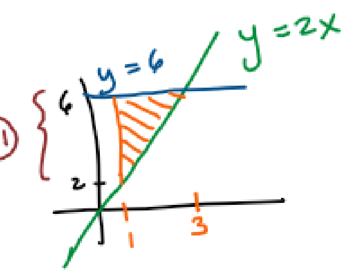
DO IT THIS WAY

$$\iint_R y\sin(xy)\,dA = \underline{\quad 1 - \frac{2}{\pi} \quad}$$

$\boxed{6}$ (b) Reverse of the order of integration ~~and evaluate~~ $\int_0^3 \int_{2x}^6 f(x,y)\,dy\,dx$. (*For full credit, you MUST draw the region*).

$\int_2^6 \int_1^{y/2} f(x,y)\,dx\,dy$

(+1) (+2)



(+1)

(+2)

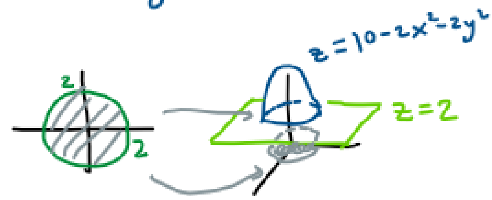
$$\int_0^3 \int_{2x}^6 \cancel{e^{y^2}}\,dy\,dx = \underline{\qquad\qquad\qquad}$$

4. (14 pts)

7 (a) Find the volume of the solid that bounded by $z = 2$ and $z = 10 - 2x^2 - 2y^2$.

(+1) I $\iint_R 10-2x^2-2y^2\,dA - \iint_R 2\,dA = \iint_R 8-2x^2-2y^2\,dA$

(+2) II $z = 10-2x^2-2y^2 \Rightarrow 2x^2+2y^2 = 8$
$\Rightarrow x^2+y^2 = 4$



(+3) III $\int_0^{2\pi}\int_0^2 (8-2r^2)\,r\,dr\,d\theta$

(+2) IV $\int_0^{2\pi}\int_0^2 8r-2r^3\,dr\,d\theta$

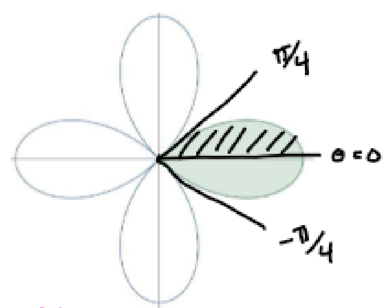
$= \int_0^{2\pi} 4r^2 - \frac{2}{4}r^4 \Big|_0^2\,d\theta$

$= \int_0^{2\pi} 16 - \frac{1}{2}\cdot 16\,d\theta$

$= 8\theta \Big|_0^{2\pi} = 16\pi$

Volume = 16π

7 (b) Use a double integral and polar to find the **area** of one loop of the rose given by $r = \cos(2\theta)$. (*shown below*)



(+1) $r = 0 \Rightarrow \cos(2\theta) = 0 \Rightarrow 2\theta = \ldots, -\frac{\pi}{2}, \frac{\pi}{2}, \ldots$
$\theta = \ldots, -\pi/4, \pi/4, \ldots$

(+1) $\iint_R 1\,dA$

(+2) $= \int_{-\pi/4}^{\pi/4}\int_0^{\cos(2\theta)} 1\cdot r\,dr\,d\theta$

(+1) $= 2\int_0^{\pi/4} \frac{1}{2}r^2\Big|_0^{\cos(2\theta)}\,d\theta$

$= \int_0^{\pi/4} \cos^2(2\theta)\,d\theta$

$= \frac{1}{2}\int_0^{\pi/4} 1+\cos(4\theta)\,d\theta$

(+2) $\frac{1}{2}\left(\theta + \frac{1}{4}\sin(4\theta)\Big|_0^{\pi/4}\right)$

$= \frac{1}{2}\left(\pi/4 + 0\right)$

$\iint_D 1\,dA = \pi/8$