12

1. (~~14~~ pts)

4

(a) Find the area of the triangle through $P(1,2,3)$, $Q(-1,3,5)$, and $R(2,1,6)$.

$\vec{i} \quad \vec{j} \quad \vec{k}$

$\overrightarrow{PQ} = \langle -2, 1, 2 \rangle$

2

$\overrightarrow{PR} = \langle 1, -1, 3 \rangle$

$= (3 - -2)\vec{i} - (-6-2)\vec{j} + (2-1)\vec{k}$

2

$= \langle 5, 8, 1 \rangle$

$\text{AREA} = \frac{1}{2}\sqrt{25+64+1}$

Area = $\boxed{\frac{1}{2}\sqrt{90} = \frac{3}{2}\sqrt{10} \approx 4.74}$

4

(b) The curves $y = x^3$ and $y = 3x + 2$ intersect at the point $(2, 8)$. Find the angle between the curves at this point of intersection. Give your final answer in degrees rounded to one decimal place.

$y' = 3x^2 \Rightarrow$ AT $x = 2$ WE HAVE $y' = 12 \Rightarrow \vec{u} = \langle 1, 12 \rangle$ IS PARALLEL TO THIS TANGENT LINE

2

$y' = 3 \Rightarrow \vec{v} = \langle 1, 3 \rangle$ IS PARALLEL TO THIS TANGENT LINE

2

$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos\theta \Rightarrow \theta = \cos^{-1}\left(\frac{1+36}{\sqrt{145}\sqrt{10}}\right) \approx 13.67°$

0.2386 RADIANS

Angle = $\boxed{13.7}$ degrees

4

(c) Consider the sphere, S, which has a diameter with endpoints at $A(0,0,1)$ and $B(6,4,3)$. Find the radius of the circular intersection of the sphere, S, with the xy-plane.

CENTER $= (3, 2, 2)$ $\qquad (x-3)^2 + (y-2)^2 + (z-2)^2 = r^2$

2

RADIUS OF SPHERE $= \sqrt{3^2 + 2^2 + (2-1)^2} = \sqrt{9+4+1} = \sqrt{14}$

INTERSECTION OF xy-PLANE $\Rightarrow (x-3)^2 + (y-2)^2 + 4 = 14$

$(x-3)^2 + (y-2)^2 = 10$

2

Radius of the circular intersection with xy-plane = $\boxed{\sqrt{10}}$

$\boxed{\approx 3.16}$

2. (14 pts)

$\boxed{2}$ (a) (2 pts) Give the precise name for the surface given by the equation $2x^2 - y^2 + z = 4$.

Surface name: HYPERBOLIC PARABOLOID

$\boxed{6}$ (b) Find parametric equations for the line of intersection of the planes $2x + y + z = 3$ and $3x + y - z = 7$.

$$3\left\{\begin{array}{l} \left.\begin{array}{l} ①\ 2x+y+z=3 \\ ②\ 3x+y-z=7 \end{array}\right\} \quad 5x+2y=10 \\ x=0 \Rightarrow y=5 \Rightarrow z=-2 \qquad A(0,5,-2) \end{array}\right.$$

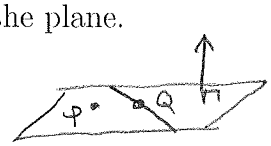
$$2\left\{\begin{array}{l} y=0 \Rightarrow x=2 \Rightarrow z=-1 \qquad B(2,0,-1) \\ \qquad\qquad\qquad\qquad \vec{AB} = \langle 2,-5,1\rangle \\ \text{NOTE: } \begin{array}{l} \langle 2,1,1\rangle \\ \times \langle 3,1,-1\rangle \end{array} \\ = (-1-1)\vec{i} - (-2-3)\vec{j} + (2-3)\vec{k} \end{array}\right.$$

$$= \langle -2, 5, -1\rangle$$

PARALLEL AND ALSO ACCEPTABLE

1 Line Equations: $x = 2t,\ y = 5-5t,\ z = -2+t$

$\boxed{6}$ (c) Find the equation of the plane that passes through the point $P(3, 4, 1)$ and contains the line $x = 5 + t$, $y = 2t - 1$, $z = -2t$. And find the y-intercept of the plane.



$$2\left\{\begin{array}{l} Q(5,-1,0) \\ \qquad\qquad \vec{i} \quad \vec{j} \quad \vec{k} \\ \vec{PQ} = \langle 2, -5, -1\rangle \end{array}\right.$$

$$2\left\{\begin{array}{l} \vec{v} = \langle 1, 2, -2\rangle \\ = (10 - -2)\vec{i} - (-4 - -1)\vec{j} + (4 - -5)\vec{k} \\ = \langle 12, 3, 9\rangle \end{array}\right.$$

$x=0,\ z=0 \Rightarrow$

$$-36 + 3(y-4) - 9 = 0$$
$$3(y-4) = 45$$
$$y-4 = 15$$

1 $\longrightarrow$ Plane Equation: $12(x-3) + 3(y-4) + 9(z-1) = 0$

1 $\longrightarrow$ y-intercept: $(x, y, z) =$ $(0, 19, 0)$

3. (12 pts) An object 'accidentally' thrown toward a certain math professor has location given by $\mathbf{r}(t) = \langle t^2, 10t, 40t - 10t^2 \rangle$ at time t seconds, where distances are in feet.

4

(a) Find parametric equations for the tangent line to the curve at the moment when the tangent line is parallel to the xy-plane.

$\vec{r}\,'(t) = \langle 2t, 10, 40 - 20t \rangle$

2 WANT DIRECTION PARALLEL TO xy-PLANE

$\Rightarrow 40 - 20t = 0 \Rightarrow t = 2$

$\vec{r}(2) = \langle 4, 20, 80 - 40 \rangle = \langle 4, 20, 40 \rangle$

$\vec{r}\,'(2) = \langle 4, 10, 0 \rangle$

2

Tangent Line Equations: $x = 4 + 4t,\ y = 20 + 10t,\ z = 40$

4

(b) Find the speed of the object at the positive time when it intersects the xy-plane.

2 $40t - 10t^2 \stackrel{?}{=} 0 \Rightarrow 10t(4 - t) = 0 \Rightarrow \begin{cases} t = 0 \\ t = 4 \end{cases}$

$\vec{r}\,'(4) = \langle 8, 10, -40 \rangle$

$|\vec{r}\,'(4)| = \sqrt{64 + 100 + 1600} = \sqrt{1764}$

2

Speed: 42 feet/sec

4

(c) Find the time at which the tangential component of acceleration, a_T, is equal to zero. You may round your final answer to two digits after the decimal.

$a_T = \dfrac{\vec{r}\,' \cdot \vec{r}\,''}{|\vec{r}\,'|} \stackrel{?}{=} 0$ $\qquad \vec{r}\,''(t) = \langle 2, 0, -20 \rangle$

2 $\Rightarrow \vec{r}\,' \cdot \vec{r}\,'' = 0 \Rightarrow 4t + 0 - 20(40 - 20t) = 0$

$4t - 800 + 400t = 0$

$404t = 800$

2 $t = \dfrac{800}{404} = \dfrac{200}{101} \approx 1.98$

$t =$ 1.98 seconds

4. (12 pts)

4

(a) Set up (but do NOT evaluate) an integral that gives the arc length of the curve of intersection of the cylinder $x^2 + y^2 = 4$ and the plane $z = 3x$. Hint: First, find parametric equations for the curve of intersection. And again don't try to evaluate the integral, just set it up with the correct bounds and integrand and write your integral below.

2 $\begin{cases} x = 2\cos(t) \rightarrow x' = -2\sin(t) \\ y = 2\sin(t) \rightarrow y' = 2\cos(t) \\ z = 6\cos(t) \rightarrow z' = -6\sin(t) \end{cases}$

2

$4\sin^2(t) + 4\cos^2(t) + 36\sin^2(t)$

Arc Length Set-Up = $\int_0^{2\pi} \sqrt{4 + 36\sin^2(t)}\, dt$

4

(b) The velocity of an object is given by $\mathbf{r}'(t) = \langle 2\sin(t), 5, te^{t^2} \rangle$ at time t seconds and its initial position is $\mathbf{r}(0) = \langle 0, 0, 1 \rangle$.

i. Find the vector function, $\mathbf{r}(t)$, for the position of this object at time t.

SUBSTITUTION

2 $\vec{r}(t) = \langle -2\cos(t) + C_1,\ 5t + C_2,\ \frac{1}{2}e^{t^2} + C_3 \rangle$

$\vec{r}(0) = \langle -2 + C_1,\ C_2,\ \frac{1}{2} + C_3 \rangle = \langle 0, 0, 1 \rangle \Rightarrow C_1 = 2, C_2 = 0$
$C_3 = \frac{1}{2}$

2

$\mathbf{r}(t) =$ $\langle -2\cos(t) + 2,\ 5t,\ \frac{1}{2}e^{t^2} + \frac{1}{2} \rangle$

4

ii. Find the curvature of the curve given by $\mathbf{r}(t)$ at $t = 0$. Hint: Plug in $t = 0$ as early as possible!

2 $\vec{r}'(t) = \langle 2\sin(t), 5, te^{t^2} \rangle$

$\vec{r}''(t) = \langle 2\cos(t), 0, e^{t^2} + 2t^2e^{t^2} \rangle$

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 5 & 0 \\ 2 & 0 & 1 \end{vmatrix}$

$(5-0)\hat{i} - (0-0)\hat{j} + (0-10)\hat{k}$

$\langle 5, 0, -10 \rangle$

$\dfrac{|\vec{r}'(0) \times \vec{r}''(0)|}{|\vec{r}'(0)|^3} = \dfrac{\sqrt{25+100}}{5^3} = \dfrac{5\sqrt{1+4}}{5^3}$

2

$\kappa(0) =$ $\dfrac{\sqrt{5}}{5^2}$