Math 126
Exam 1
April 25, 2024

Name _____

Student ID #_____

Section _____

HONOR STATEMENT

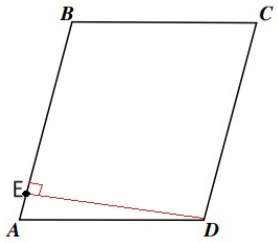
"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam."

SIGNATURE:_____

- This exam consists of this cover, four pages of questions, and a blank "scratch sheet". If you put work on the scratch sheet and you want it to be graded, then you must clearly tell us in the problem to "see scratch page".
- You will have 50 minutes.
- You are allowed to use a Ti-30x IIS Calculator model ONLY (**no other calculators allowed**) and one 8.5 by 11 inch sheet of handwritten notes (front and back). All other sources are forbidden.
- Turn your cell phone OFF and put it away for the duration of the exam. You may not listen to headphones or earbuds during the exam.
- **You must show your work.** The correct answer with no supporting work may result in no credit.
- Leave your answer in exact form. Simplify standard trig, inverse trig, natural logarithm, and root values. Here are several examples: you should write $\sqrt{4} = 2$ and $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ and $\ln(1) = 0$ and $\tan^{-1}(1) = \frac{\pi}{4}$.
- Unless otherwise indicated, when rounding is necessary, you may round your final answer to two digits after the decimal.
- **Do not write within 1 centimeter of the edge!** Your exam will be scanned for grading.
- There may be multiple versions, you have signed an honor statement, and cheating is a hassle for everyone involved. If we find that you give an answer that is only appropriate for the other version of the exam and there is no work to support your answer, then you will get a zero on the entire exam and your work will be submitted to the academic misconduct board. JUST DO NOT CHEAT.

GOOD LUCK!

1. (14 pts) The parallelogram below has corners at the points A, B, C and D. The point E is discussed in the last question on this page. You are given the following information:



- The location of the points $A(1, 2, 3)$ and $B(2, 3, 5)$.
- The vector $\overrightarrow{AD} = \langle 3, -1, 0 \rangle$

(a) Give the (x, y, z) coordinates of the point C.

$(x, y, z) =$ _____

(b) Find the area of the parallelogram and give the equation of the plane containing this parallelogram.

Area = _____

Plane Equation = _____

(c) Find the angle $\angle BAD$, which is the angle at the vertex A.
(*Give your final answer in degrees, rounded to the nearest degree*)

Angle = _____ degrees

(d) The point E is on the line segment from A to B and the line from E to D is perpendicular to the line segment from A to B (as shown). Find the vector $\overrightarrow{DE}$.

$\overrightarrow{DE}$ vector: _____

2. (12 pts)

(a) Determine whether each statement is true or false in $\mathbb{R}^3$.
(Put "$\times$" in the circle next to your choice)

i. ◯ TRUE ◯ FALSE: Two different lines parallel to a given plane must be parallel.

ii. ◯ TRUE ◯ FALSE: Two different planes orthogonal to a given line must be parallel.

(b) Consider the line L_1 given by $x = 9 + t$, $y = 4 + 2t$, $z = 1 - 5t$. A second line, L_2, is perpendicular to the plane $3x - y + 5z = 30$ and intersects this plane at its z-intercept. Give parametric equations for the line L_2 and find the intersection of the two lines L_1 and L_2 (*if they do not intersect, then write DNE*).

Line Equations for L_2: _____

Intersection of L_1 and L_2: $(x, y, z) =$ _____

(c) Find the equation of the plane that passes through the point $(3, 2, 1)$ and contains the line of intersection of the two planes $2x + y + 5z = 9$ and $x - y + z = 3$.

Plane Equation: _____

3. (12 pts) Consider the curve given by $\mathbf{r}(t) = \langle \sqrt{4t+1}, e^{(t^2-4)}, t^3 \rangle$.

(a) Find parametric equations for the tangent line to the curve at the point $(3, 1, 8)$. And give the point of intersection of this tangent line with the plane $3x - 2y + z = 33$.

Intersection point $(x, y, z) =$ _____

(b) Find the arc length of the curve of intersection of the elliptical cylinder $4x^2 + y^2 = 4$ and the plane $z = \sqrt{3}x$. *(Parameterize AND compute the arc length integral)*

Length = _____

4. (12 pts)

(a) Find and simplify an equation for the surface consisting of all points P whose distance to the y-axis is 5 times the distance to the plane $y = 1$. Then give the precise name of this surface.

Equation: _____

Name: _____

(b) Find all constants, a, so that the curve $\mathbf{r}(t) = \langle 2\cos(t), 2\sin(t), at^2 \rangle$ has a curvature of $\kappa(0) = 1$ at $t = 0$.

$a =$ _____

(c) Dr. Loveless throws an object into the air. Gravity and wind act on the object as it moves. The acceleration of the object is given by $\mathbf{a}(t) = \langle e^{-t}, 0, -10 \rangle$.
The initial velocity is $\mathbf{v}(0) = \langle 0, 4, 10 \rangle$ and the initial position is $\mathbf{r}(0) = \langle 0, 0, 15 \rangle$. Find the positive time when the object hits the xy-plane and give the velocity vector at this time.

velocity vector when object hits xy-plane: _____

You may use this page for scratch-work or extra room.
All work on this page will be ignored unless you write and circle "see scratch page" on the problem page and you label your work below.