Exam 2 Review

Exam 2 will cover 6.4, 6.5, 7.1 - 7.5, 7.7, 7.8, and 8.1. You are also expected to have a basic under-
standing of the material that was covered on Exam 1. This review sheet discusses, in a very basic way,
the key concepts from these sections. This review is not meant to be all inclusive, but hopefully it
reminds you of some of the basics. Please notify me if you find any typos in this this review.

If you understand the concepts behind all the homework problems and if you can complete them quickly
and competely, then you will do well on the exam. The exam will test you on the same concepts and
ideas as the homework problems. You should also take a look at old exams (specifically written by
Bube, Pollack, Conroy, and Perkins).

1. A Chronological Review

- Section 6.4 (WORK): Understand how to think through work problems. You need to know
  how to use the following : Work $= \int_a^b$ Force dx. More specifically, you need to understand
  the following three cases:

  Work to Stretch a Spring $= \int_a^b kx \, dx$
      where $x =$ distance beyond natural length, and $k =$ spring constant.
  Work to Lift a Cable $= \int_a^b kx \, dx$
      where $x =$ distance below lifting point, and $k =$ weight of cable per foot.
  Work to Pump Water to a Given Height $=$ (weight of water) $\int_a^b$ (Area of a Horiz. Slice) $x \, dx$
      where $x =$ distance below the pumping height, and
      weight of water $=$ (density)(gravity) $= 9800$ N/m$^3 = 62.5$ lb/ft^3

- Section 6.5 (AVERAGE VALUE): Understand how to answer questions involving the average
  value of a function.

$$\text{Average of } f(x) \text{ from } x = a \text{ to } x = b \ = \ \frac{1}{b-a} \int_a^b f(x) \, dx$$

- Section 7.1 (INTEGRATION BY PARTS): Be able to identify when to use integration by
  parts and be able to use it. It is good for problems involving simple products, especially
  products involving $\ln(x)$ and inverse trigonometric functions. Remember how to choose your
  u and use the equation: $\int u \, dv = uv - \int v \, du$.

- Section 7.2 (TRIGONOMETRIC INTEGRALS): Be able to work through the 5 cases we
  discussed in class. Specially note the common theme of setting up a u-substitution. The
  only case where we don't set up a u-substitution is when the powers on $\sin(x)$ and $\cos(x)$
  are both even (then we use the half angle identities).

  - Odd power on $\sin(x)$: Pull out one factor of $\sin(x)$, use the identity $\sin^2(x) = 1-\cos^2(x)$,
    then let $u = \cos(x)$ and perform the u-substitution.
  - Odd power on $\cos(x)$: Pull out one factor of $\cos(x)$, use the identity $\cos^2(x) = 1-\sin^2(x)$,
    then let $u = \sin(x)$ and perform the u-substitution.
  - Even power on $\sec(x)$: Pull out one factor of $\sec^2(x)$, use the identity $\sec^2(x) = 1 +
    \tan^2(x)$, then let $u = \tan(x)$ and perform the u-substitution.
  - Odd power on $\tan(x)$: Pull out one factor of $\sec(x)\tan(x)$, use the identity $\tan^2(x) =
    1 - \sec^2(x)$, then let $u = \sec(x)$ and perform the u-substitution.
  - Even power on both $\sin(x)$ and $\cos(x)$: Use the half angle identities (they are in your
    book and on a previous review sheet).

  In this section we also discussed the integrals:

$$\int \tan(x) \, dx = \ln|\sec(x)| + C \qquad \text{and} \qquad \int \sec(x) \, dx = \ln|\sec(x) + \tan(x)| + C$$

  Finally, you may need to think on your feet here. When in doubt you may want to rewrite
  $\sec(x)$ as $1/\cos(x)$, or $\tan(x)$ as $\sin(x)/\cos(x)$.

- Section 7.3 (TRIGONOMETRIC SUBSTITUTION): These problems are long, but systematic. Make sure you know the steps:

  (a) Complete the square if there is a 'middle' term.

  (b) Perform the trig. substitution. You will have to decide whether $x = a\sin(\theta)$, $x = a\tan(\theta)$, or $x = a\sec(\theta)$. Then you alway do the next three things: 1) Compute dx, 2) Substitute for x and dx, and 3) Use an identity to simplify the squared part $(1 - \sin^2(\theta) = \cos^2(\theta)$, $1 + \tan^2(\theta) = \sec^2(\theta)$, and $\sec^2(\theta) - 1 = \tan^2(\theta))$.

  (c) Evaluate the Trig. Integral. When you simplify the integral from the previous step, it will usually have trig functions in it. To evaluate such an integral you will have to use 7.2 material.

  (d) Return to x. Draw you triangle. This allows you to go from trig function to trig function which is essential since the only relationship you have between x and θ is a trig function.

- Section 7.4 (PARTIAL FRACTIONS): Be able to integrate rational functions for which the denominator factors.

  (a) If the degree (largest power) on top is bigger than, or equal to, the degree on bottom, DIVIDE.

  (b) Factor the bottom and decompose into partial fractions. There are two cases you must know: distinct linear factors and non-distinct linear factors.

- Section 7.5 (INTEGRATION STRATEGIES): You need to know how to choose the right integration technique. One good strategy is to work lots of problems. It would also be a good idea to take the old exams which have posted solutions (For each integral on these exams, write down in words how you would start the problem and perform the first step. In this way, you can look at lots of problems. Then look at the solutions and see if you are headed in the right direction).

  I have posted two integration reviews already, but here is a quick reminder:

  (a) First look for obvious simplifications or substitutions.

  (b) Then try to use a process of eliminations between the four major techniques
    - products / logs / inverse trig $\Rightarrow$ **Try Integration by Parts**
    - $\sin(x)$ / $\cos(x)$ / $\tan(x)$ / $\sec(x)$ $\Rightarrow$ **Try Trigonometric Integrals**
    - BLAH $-x^2$, x^2- BLAH, x^2+ BLAH, or quadratic that doesn't factor, or square root $\Rightarrow$ **Try Trigonometric Substitution**
    - rational function where bottom factors $\Rightarrow$ **Try Partial Fractions**

  (c) If none of these methods appears obvious, then you might need to try a u-substution (it is often smart to try $u = $ a trig function, a power of x, or a square root).

- Section 7.7 (APPROXIMATING INTEGRALS): You should know the Midpoint Rule, Trapezoid Rule, and Simpson's Rule well. Specifically, understand how to compute Δx and then break up your interval to find x_0, x_1, *etc*. Then you should be able to use the formulas. On a test you will not be asked to evaluate the sum (that is you don't have to plug into your calculator to get a numerical answer), but you do need to know how to set these up.

- Section 7.8 (IMPROPER INTEGRALS): Be able to identify infinite and discontinuous integrals. YOUR FIRST STEP IS ALWAYS TO REWRITE THE INTEGRAL AS A LIMIT (YOU MUST DO THIS)!!! Then evaluate the integral and finally evaluate the limit.

- Section 8.1 (ARC LENGTH): Understand how to set up and evaluate an integral to find the arc length of a curve.

$$\text{Arc Length } = \int_a^b \sqrt{1 + [f'(x)]^2} \, dx$$

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